SMT359

Glossary for all books

Numbers in parentheses are page references with B1, B2, B3 denoting Book 1 (*An introduction to Maxwell's equations*), Book 2 (*Electromagnetic fields*) and Book 3 (*Electromagnetic waves*), respectively.

Italicized words are cross-references to other entries in this Glossary.

4-vector (B2: 222) A vector with four components that transforms between different *inertial frames* of reference according to the *Lorentz transformation* matrix. The *coordinates* of an *event* (ct, x, y, z), and the combination of *charge density* and *current density* $(c\rho, J_x, J_y, J_z)$ are both 4-vectors.

absorption (B3: 107) (of electromagnetic radiation) The transformation of the energy of an electromagnetic wave into different forms. In the simple classical model of a *dielectric material*, absorption is due to 'frictional' forces on *bound electrons* set in motion by the electromagnetic wave. Absorption in the infrared region is due to excitation of oscillations of *permanent electric dipoles*.

absorption length (B3: 108) The distance over which the *amplitude* of an electromagnetic wave is reduced by a factor of $\exp(-1) = 0.37$ through damping.

action at a distance See *instantaneous action at a distance*.

addition of forces (B1: 20, 71) If a particle is acted on by a number of different forces, it responds as if it experienced a single force equal to the total or resultant force. The total force is obtained by forming the vector sum of all the forces acting on the particle.

additivity of charge (B1: 13) The total *charge* within a given region is the sum of all the charges in the region, with due account taken of the signs of the charges.

additivity of circulation (B1: 232) The *circulation* of a *vector field* around the perimeter of an *open surface* is the sum of its circulations around the perimeters of its subareas.

The additivity of circulation allows us to introduce the concept of *curl* and establish the *curl theorem*. Because the *current* crossing a surface is additive, the additivity of *magnetic circulation* is needed to ensure the internal consistency of *Ampère's law*.

additivity of flux (B1: 223) The *flux* of a *vector field* over the surface of a region is the sum of the fluxes over the surfaces of its subregions.

The additivity of flux allows us to introduce the concept of *divergence* and establish the *divergence theorem*. Because charge is additive, the additivity of *electric flux* is needed to ensure the internal consistency of *Gauss's law*.

algebraic sum (B1: 13) A sum of algebraic quantities, with due account taken of their signs.

alternator (B1: 153) An *electric generator* that produces an alternating current.

ampere (B1: 62, 69) The SI unit of *current* (symbol A). The ampere is defined as that steady current which, when carried by two parallel wires separated by a metre in a vacuum, causes each wire to experience a *magnetic force* per unit length of $2 \times 10^{-7}\,\mathrm{N\,m^{-1}}$. When a current of one ampere flows along a wire for one second, a charge of one *coulomb* passes any fixed point on the wire.

Ampère–Maxwell law (B1: 170) A modification of *Ampère's law* which achieves consistency with the *equation of continuity*. The differential version of this law is

$$\mathrm{curl}\,\mathbf{B} = \mu_0\mathbf{J} + \varepsilon_0\mu_0\frac{\partial \mathbf{E}}{\partial t},$$

where $\mathbf{B}$ is the *magnetic field*, $\mathbf{E}$ is the *electric field*, $\mathbf{J}$ is the *current density*, ε_0 is the *permittivity of free space* and μ_0 is the *permeability of free space*.

The second term on the right-hand side is called the *Maxwell term*. The quantity $\varepsilon_0 \partial\mathbf{E}/\partial t$ is often called the *displacement current density*.

The integral version of the Ampère–Maxwell law is

$$\oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 \int_S \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \cdot \mathrm{d}\mathbf{S},$$

where S is any *open surface* and C is its perimeter. The Ampère–Maxwell law is one of *Maxwell's equations* of electromagnetism.

In the presence of magnetic and dielectric materials, it is often more convenient to express the Ampère–Maxwell law in terms of the *magnetic intensity* $\mathbf{H}$, *the electric displacement* $\mathbf{D}$ and the *free current* density $\mathbf{J}_\mathrm{f}$:

$$\oint_C \mathbf{H} \cdot \mathrm{d}\mathbf{l} = \int_S \left(\mathbf{J}_\mathrm{f} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot \mathrm{d}\mathbf{S},$$

$$\operatorname{curl} \mathbf{H} = \mathbf{J}_\mathrm{f} + \frac{\partial \mathbf{D}}{\partial t}.$$

Ampère's circuital law An alternative term for the integral version of *Ampère's law*.

Ampère's law (B1: 94, 111) This law of *magnetostatics* has both integral and differential versions.

The integral version of Ampère's law states that the *circulation* of the *magnetic field* around a *closed loop* C is equal to the steady *current* through any *open surface* S bounded by the loop, multiplied by μ_0, the *permeability of free space*. That is,

$$\oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 \int_S \mathbf{J} \cdot \mathrm{d}\mathbf{S}.$$

The *curl theorem* leads to an equivalent differential version. This states that the *curl* of the magnetic field at any point in space and instant in time is equal to μ_0 times the *current density* at the same point and instant:

$$\operatorname{curl} \mathbf{B} = \mu_0 \mathbf{J}.$$

Ampère's law applies only to steady currents and the magnetic fields they produce.

In the presence of magnetic materials, it is often more convenient to express Ampère's law in terms of the *magnetic intensity* $\mathbf{H}$ and the *free current* density $\mathbf{J}_\mathrm{f}$:

$$\oint_C \mathbf{H} \cdot \mathrm{d}\mathbf{l} = \int_S \mathbf{J}_\mathrm{f} \cdot \mathrm{d}\mathbf{S},$$

$$\operatorname{curl} \mathbf{H} = \mathbf{J}_\mathrm{f}.$$

amplitude (B1: 182, B3: 15) The maximum value of a sinusoidal waveform. For the function $A(z, t) = A_0 \cos(kz - \omega t + \phi)$, the amplitude is A_0.

amplitude reflection ratio (B3: 74) (of an electromagnetic wave) The ratio of the *amplitude* of the electric field of the wave reflected from a boundary to the amplitude of the electric field of the incident wave. For reflection from a boundary between two *dielectric materials*, the amplitude reflection ratio is determined by the *Fresnel equations*.

amplitude transmission ratio (B3: 74) (of an electromagnetic wave) The ratio of the *amplitude* of the electric field of the wave transmitted across a boundary to the amplitude of the electric field of the incident wave. For transmission across a boundary between two *dielectric materials*, the amplitude transmission ratio is determined by the *Fresnel equations*.

angular dispersion (B3: 98) The splitting of electromagnetic radiation into its spectral components at an interface between *dielectric materials*. Angular dispersion is a consequence of the dependence of the speed of propagation on the *frequency* of the radiation, which is described by a frequency-dependent *refractive index* $n(\omega)$.

angular frequency (B1: 182, B3: 15) The product of the *frequency* (number of cycles per unit time) of a periodic quantity and the factor 2π. Angular frequency is denoted by ω and measured in s^{-1}.

anode (B2: 119) A positive electrode. In the *electron gun* in devices like the cathode ray tube or electron microscope, electrons emitted from the *cathode* are accelerated to high energies by being attracted towards the anode.

areal charge density (B1: 53) The *charge* per unit area around a given point on a surface or a thin plate. The SI unit of areal density is $\mathrm{C\,m^{-2}}$. If only a single surface is involved, then areal charge density is equivalent to *surface charge density*.

arrow map (B1: 204) A way of representing a *vector field* in a given region of space. An arrow map consists of arrows with their tails at a selection of points in the region. At each of these points, the direction of the arrow is the direction of the field, and the length of the arrow is proportional to the *magnitude* of the field. Contrast with *field line pattern*.

axial coordinate (B1: 210) One of the *cylindrical coordinates*. The axial coordinate, z, of a point is the usual *Cartesian coordinate*, measured along the z-axis.

axial symmetry (B1: 32) An object or a *field* is said to have axial symmetry about a given axis if it is unchanged when rotated through any angle about the axis.

axis of polarization (B3: 84) When *linearly polarized* light passes through a polarizing filter, the attenuation has its minimum value when the

polarization direction is aligned with the axis of polarization of the filter. Light that is polarized perpendicular to the axis of polarization is attenuated most strongly.

azimuthal coordinate (B1: 205, 210) In *spherical coordinates*, the azimuthal coordinate, ϕ, of a point is the angle between the x-axis and the projection in the xy-plane of the line joining the origin to the point. In *cylindrical coordinates*, the azimuthal coordinate, ϕ, of a point is the angle between the x-axis and the shortest line joining the z-axis to the point. In both cases, the sense of increasing ϕ is determined by a *right-hand grip rule*: with the thumb of the right hand pointing along the positive z-axis, the curled fingers of the right hand indicate the direction in which ϕ increases.

back-emf (B2: 163) The *emf* induced in a circuit due to a change in current through that circuit is known as a back-emf because it acts to oppose changes in the current. The relationship between this induced back-emf and the current is $V_{\text{emf}} = -L\,\mathrm{d}I/\mathrm{d}t$, which indicates that an increase in current leads to a negative emf, which opposes the increase.

Biot–Savart field law (B1: 71) for a *current element*. The *magnetic field* due to a steady *current element* $I\,\delta\mathbf{l}$ at the point $\mathbf{r}_0$ is

$$\delta\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\,\frac{I\,\delta\mathbf{l}\times(\mathbf{r}-\mathbf{r}_0)}{|\mathbf{r}-\mathbf{r}_0|^3},$$

where $\mathbf{r}$ is the point at which the field is measured and μ_0 is the *permeability of free space*. The Biot–Savart field law does not apply to current elements that change in time.

Biot–Savart force law (B1: 66) for *current elements*. Given two steady current elements, $I_1\,\delta\mathbf{l}_1$ and $I_2\,\delta\mathbf{l}_2$ at points $\mathbf{r}_1$ and $\mathbf{r}_2$, the *magnetic force* on the first current element due to the second is

$$\delta\mathbf{F}_{12} = \frac{\mu_0}{4\pi}\,\frac{I_1\delta\mathbf{l}_1\times(I_2\delta\mathbf{l}_2\times\widehat{\mathbf{r}}_{12})}{r_{12}^2},$$

where μ_0 is the *permeability of free space*.

The Biot–Savart force law is an inverse square law with a numerator that involves *vector products*. The direction of the force is in the plane of $I_2\,\delta\mathbf{l}_2$ and $\widehat{\mathbf{r}}_{12}$, and is perpendicular to $I_1\,\delta\mathbf{l}_1$ in a sense determined by the *right-hand rule*. This law of force between current elements does not obey Newton's third law, although the force on one complete circuit due to another complete circuit does obey Newton's third law. The Biot–Savart force law does not apply to current elements that change in time.

Biot–Savart law This could refer to the *Biot–Savart force law* or the *Biot–Savart field law*.

birefringence (B3: 88) The property of certain *dielectric materials*, with anisotropic crystal structures, of having two different speeds of wave propagation for two specific planes of *polarization*. This means that two refracted rays are produced from a single incident ray.

Boltzmann constant (B2: 22, B3: 149) A fundamental constant, denoted k_B, that relates thermal energy U to absolute temperature T, usually as $U \sim k_\text{B}T$. Its value is $k_\text{B} = 1.38\times10^{-23}\,\mathrm{J\,K^{-1}}$.

bound charge (B2: 17) The charge within a material that is unable to move freely through the material. Small displacements of bound charge are responsible for *polarization* of a material by an electric field. Compare with *free charge*.

bound current (B2: 50) A current that is the net effect of atomic-scale currents in the atoms of a magnetized object. See *bound surface current* and *bound current density*.

bound current density (B2: 52) The macroscopic *current density* in the bulk of a magnetic material resulting from the microscopic currents associated with *magnetic dipoles*. The bound current density at a point is given by $\mathbf{J}_\text{b} = \operatorname{curl}\mathbf{M}$, where $\mathbf{M}$ is the local *magnetization*.

bound surface charge density (B2: 26) The charge on the surface of a polarized object due to the *polarization* of the material. A polarized object has a surface charge density σ_b that is related to the local polarization $\mathbf{P}$ by $\sigma_\text{b} = \mathbf{P}\cdot\widehat{\mathbf{n}}$, where $\widehat{\mathbf{n}}$ is a unit vector in the direction of the outward normal to the surface. The SI unit of bound surface charge density is $\mathrm{C\,m^{-2}}$.

bound surface current (B2: 50) The current flowing on the surface of a magnetized object due to the microscopic currents associated with *magnetic dipoles* in the material. A magnetized object has a surface current per unit length $\mathbf{i}_\text{b}$ that is related to the local magnetization $\mathbf{M}$ by $\mathbf{i}_\text{b} = \mathbf{M}\times\widehat{\mathbf{n}}$, where $\widehat{\mathbf{n}}$ is a unit vector in the direction of the outward normal to the surface. Thus the magnitude of $\mathbf{i}_\text{b}$ is numerically equal to M, and its direction is given by a right-hand rule; i_b is the current per unit length of surface in the direction perpendicular to the current, and its SI unit is $\mathrm{A\,m^{-1}}$.

bound volume charge density (B2: 28) The macroscopic charge density within the volume of a *dielectric* due to non-uniform *polarization* of the material. In general, a dielectric will have a bound charge density given by $\rho_\text{b} = -\operatorname{div}\mathbf{P}$, where $\mathbf{P}$ is the polarization.

boundary conditions for $\mathbf{B}$ and $\mathbf{H}$ (B2: 58, B3: 71) The fields $\mathbf{B}$ and $\mathbf{H}$ on either side of a boundary between materials 1 and 2 obey the conditions $B_{1\perp} = B_{2\perp}$ and $H_{1\parallel} = H_{2\parallel}$. Thus the normal component of the magnetic field $\mathbf{B}$ and the tangential component of the magnetic intensity $\mathbf{H}$ are the same on both sides of the boundary.

boundary conditions for current density (B2: 151) At the interface between two materials, labelled 1 and 2, the boundary conditions for the components of current density $\mathbf{J}$ are $J_{1\perp} = J_{2\perp}$ and

$$\frac{J_{1\parallel}}{\sigma_1} = \frac{J_{2\parallel}}{\sigma_2},$$

where σ_1 and σ_2 are the conductivities of the materials. The first condition is a consequence of the equation of continuity, and the second is a consequence of continuity of $E_\parallel$ across the boundary, which in turn is a consequence of Faraday's law.

boundary conditions for $\mathbf{D}$ and $\mathbf{E}$ (B2: 34, B3: 71) The fields $\mathbf{D}$ and $\mathbf{E}$ on either side of a boundary between materials 1 and 2 obey the conditions $D_{1\perp} = D_{2\perp}$ and $E_{1\parallel} = E_{2\parallel}$. So the normal component of the electric displacement $\mathbf{D}$ and the tangential component of the electric field $\mathbf{E}$ are the same on both sides of the boundary. If there is *free charge* density σ_f on the boundary, then the second condition is modified to $D_{2\perp} - D_{1\perp} = \sigma_\mathrm{f}$.

boundary conditions for electrostatic potential (B2: 78) At the interface between two materials, labelled 1 and 2, the boundary conditions for the electrostatic potential are $V_1 = V_2$ and

$$\varepsilon_1\varepsilon_0\left[\operatorname{grad} V_1\right]_\perp - \varepsilon_2\varepsilon_0\left[\operatorname{grad} V_2\right]_\perp = \sigma_\mathrm{f},$$

where ε_1 and ε_2 are the relative permittivities of the materials, and σ_f is the free surface charge per unit area at the interface.

boundary conditions for perfect conductor (B3: 131) For perfect conductors, the *skin depth* is zero, so electric and magnetic fields do not penetrate below the surface. Thus $\mathbf{E} = \mathbf{0}$ and $\mathbf{B} = \mathbf{0}$ inside a perfect conductor. The boundary conditions for the fields outside the surface are

$$E_\parallel = 0,\ B_\perp = 0,\ D_\perp = \sigma_\mathrm{f},\ H_\parallel = i_\mathrm{S},$$

where i_S is the surface current per unit length flowing perpendicular to $H_\parallel$.

breakdown field (B1: 34) The minimum *electric field strength* needed to transform an *insulator* into a *conductor*.

Brewster angle (B3: 83) When an electromagnetic wave that is polarized in the *scattering plane* is incident on a plane *dielectric* boundary at the Brewster angle θ_B, the wave is totally transmitted; there is no reflected wave. So when unpolarized light is incident at the Brewster angle, the reflected light is polarized normal to the scattering plane. If the boundary separates dielectrics with *refractive indices* n_1 and n_2, then $\tan\theta_\mathrm{B} = n_2/n_1$.

capacitance (B1: 133) The capacitance of an isolated *conductor* is the ratio Q/V, where Q is the *charge* on the conductor and V is the *potential* of the conductor relative to a zero of potential at infinity. The capacitance of a *capacitor* is the ratio Q/V, where Q is the charge on the positive plate of the capacitor and V is the *potential difference* between the positive and negative plates. Capacitance is measured in *farads* (F), so $1\,\mathrm{F} \equiv 1\,\mathrm{C}/1\,\mathrm{V}$.

capacitor (B1: 54, 133) A device used to store electrical energy by keeping positive and negative *charges* separated.

Cartesian components (B1: 195) The components of a *vector* in a *Cartesian coordinate system*. Often just called components.

Cartesian coordinate system (B1: 194) A set of three mutually perpendicular axes pointing outwards from a single origin. The axes are called the x-axis, the y-axis and the z-axis. They are usually chosen to give a *right-handed coordinate system*.

Cartesian coordinates (B1: 197) The coordinates used to describe the position of a point in a *Cartesian coordinate system*. Often just called coordinates.

Cartesian unit vectors (B1: 194) The three *unit vectors* $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ that point in the directions of the axes of a *Cartesian coordinate system*.

cathode (B2: 119) A negative electrode. In the *electron gun* in devices like the cathode ray tube or electron microscope, electrons are emitted from the cathode and are attracted towards the *anode*.

chain rule of partial differentiation (B1: 214) Suppose that $f(x, y, z)$ is a function of independent variables x, y and z. If we make arbitrary small changes δx, δy and δz to the independent variables, the function f changes by an amount δf, where

$$\delta f = \frac{\partial f}{\partial x}\,\delta x + \frac{\partial f}{\partial y}\,\delta y + \frac{\partial f}{\partial z}\,\delta z.$$

This equation is only exact in the limit as δx, δy and δz tend to zero, but it can be made as accurate as we like by taking δx, δy and δz to be small enough.

charge Used as a shorthand for *electric charge* or for a *point charge*.

charge density (B1: 38) The *charge* per unit volume around a given point $\mathbf{r}$, denoted by $\rho(\mathbf{r})$. The total charge within a volume V is given by the *volume integral*

$$Q = \int_V \rho(\mathbf{r})\,\mathrm{d}V.$$

The SI unit of charge density is $\mathrm{C\,m^{-3}}$.

chromatic aberration (B3: 101) The blurring of an image formed by a lens due to light of different colours that originates from a point on the object being focused at different distances from the lens. Chromatic aberration arises because the *refractive index* of glass depends on *frequency*, and therefore the

focal length of a lens is slightly different for different frequencies. Chromatic aberration can be reduced by using *compound lenses*.

circularly polarized (B3: 33) An electromagnetic wave of *angular frequency* ω is said to be circularly polarized if the field $\mathbf{E}$ has constant magnitude normal to the direction of propagation, but rotates about that direction with angular frequency ω. The $\mathbf{B}$ field behaves in the same way, and is always orthogonal to the $\mathbf{E}$ field. Circularly polarized waves can be considered to be the superposition of two waves with the same *frequency* and *amplitude*, but with orthogonal electric field vectors and a *phase* difference of $\pi/2$. See *right-handed (RH) circular polarization* and *left-handed (LH) circular polarization*.

circulation of a vector field (B1: 229) The *line integral* of a *vector field* $\mathbf{F}(\mathbf{r})$ around a given *closed loop* C. This is denoted by

$$\oint_C \mathbf{F}(\mathbf{r}) \cdot \mathrm{d}\mathbf{l},$$

where the circle through the integral sign indicates that C is a *closed loop*. See also *additivity of circulation*.

closed loop (B1: 229) A *directed curve* that closes on itself so that its end-point is the same as its start-point.

closed surface (B1: 40, 222) A surface that forms a complete boundary between the region inside the surface and the region outside the surface. It is not possible to pass from the interior to the exterior (or vice versa) without crossing the surface. Contrast with *open surface*.

coercivity (B2: 56) The magnitude H of the field that has to be applied to a *ferromagnetic material* previously magnetized to saturation in order to reduce the magnitude B of the internal field to zero. Material for permanent magnets should have a large coercivity; material for magnetic storage should have a moderate coercivity so that data can be rewritten.

coherence length (B2: 193) The characteristic distance over which the *number density* of *superconducting electrons* changes. It is denoted by the symbol ξ (the Greek lower-case xi, pronounced ‘ksye’).

collagen fibrils (B3: 175) Narrow strands of insoluble fibrous protein that make up a large part of skin, tendons and bone, and the *cornea*.

collimation (B2: 125) The process of converting a diverging beam (of electromagnetic radiation, charged particles, etc.) into a parallel beam. Electron beam collimators use electric or magnetic fields, and light collimators use optical lenses.

component of a vector (B1: 195) In a *Cartesian coordinate system* any *vector* can be expressed in the form

$$\mathbf{a} = a_x\mathbf{e}_x + a_y\mathbf{e}_y + a_z\mathbf{e}_z,$$

where $\mathbf{e}_x$, $\mathbf{e}_x$ and $\mathbf{e}_x$ are *Cartesian unit vectors*. The *scalar quantities* a_x, a_y and a_z are the components of the vector. They have the same units as the vector, and may be positive, negative or zero. The components depend on the directions of the coordinate axes.

compound lens (B3: 101) A composite lens consisting of two or more lenses made from materials with different *refractive indices*. Compound lenses can be designed so that the *chromatic aberrations* of the individual lenses tend to cancel.

condensate (B2: 193) The free electrons in a *superconductor* that have ‘condensed’ into a macroscopic quantum state. The condensation is a result of an attractive interaction, mediated by vibrations of the lattice, which couples together pairs of electrons.

conduction electrons (B1: 64) Electrons that have become detached from their atoms and are free to roam throughout a conductor. They are also referred to as free electrons. These electrons are the charge carriers responsible for *electric currents* in metals.

conductivity (B2: 142) The conductivity σ of a material is defined by $\mathbf{J} = \sigma\mathbf{E}$, where $\mathbf{J}$ is the *current density* and $\mathbf{E}$ is the electric field. This definition assumes that there is no magnetic force on the charge carriers, that is, that the material is stationary and/or the external magnetic field is zero. More generally, $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, and this relationship is the local form of *Ohm’s law*. The SI unit of conductivity is $\Omega^{-1}\,\mathrm{m}^{-1}$.

conductor (B1: 64) A material (for example a metal) that is able to conduct *electric currents*.

conservation of charge (B1: 13, 167) *Charge* is conserved globally, so the total charge of the Universe remains constant. Charge is also conserved locally, so the total charge in any region of space, remains constant unless charged particles flow across the boundary of the region. See also *equation of continuity*.

conservative electric field (B1: 115) An *electric field* that is conservative. *Electrostatic* fields are conservative, but electric fields produced by *electromagnetic induction* are non-conservative.

conservative vector field (B1: 117, 238) A *vector field* $\mathbf{F}$ is said to be conservative in a region R if it has any of the following properties:

- The *circulation* of $\mathbf{F}$ around any *closed loop* in R is equal to zero.
- Any *line integral* of $\mathbf{F}$ in R depends only on its start-point and end-point.

- Any line integral of $\mathbf{F}$ in R can be expressed as minus the difference in values of a scalar field between the end-point and start-point of the path.
- There is a scalar field f such that $\mathbf{F} = -\operatorname{grad} f$ throughout R.

These properties are all true for conservative fields and they are all false for non-conservative fields. The scalar field f is called the *scalar potential*.

In addition, every conservative field in R is *irrotational* in R. However, knowledge that a vector field is irrotational in R does not guarantee that it is conservative in R unless R is *simply-connected*. See also *curl test*.

constant field This generally means a *field* that does not vary in time. Also referred to as a *steady field*.

contour lines (B1: 204) A way of visualizing a *scalar field* in two dimensions. Each contour line joins points with a fixed value of the field.

contour surfaces (B1: 204) A way of visualizing a *scalar field* in three dimensions. Each contour surface joins points with a fixed value of the field. For a field of *electrostatic potential*, the contour surfaces are called *equipotential surfaces*.

convex surface (B1: 41) A *closed surface* that bulges outwards everywhere so that, viewed from the outside, there are no hollows. *Gauss's law* is most easily established for convex surfaces, but is actually true for all closed surfaces.

coordinate-free expression (B3: 26) An expression involving spatially-dependent quantities that does not explicitly refer to the coordinates in a specific coordinate system. For example, the relationship $\mathbf{E} = c\,\mathbf{B} \times \widehat{\mathbf{k}}$ is coordinate-free, but $\mathbf{E}(z,t) = c\mathbf{B}(z,t) \times \mathbf{e}_z$ is not.

coordinates (of an event) (B2: 220) The four quantities (ct, x, y, z) used to specify the location of an *event* in time and space according to some particular *frame of reference*.

cornea (B3: 175) The transparent region through which light enters the eye. The cornea protects the interior of the eye from damage, and it acts as a fixed-focus lens, which, in combination with the variable-focus lens within the eye, forms images on the retina.

coulomb (B1: 19, 69) The SI unit of *electric charge*, denoted by the symbol C. The coulomb is formally defined as the charge transferred by a steady *current* of one *ampere* in a time interval of one second, so $1\,\mathrm{C} = 1\,\mathrm{A}/1\,\mathrm{s}$.

Coulomb's law (B1: 18) The *electrostatic force* between two stationary *point charges* acts along their line of separation. This force is repulsive for charges of the same sign and attractive for charges of opposite sign. The *magnitude* of the force is proportional to the product of the charges and inversely proportional to the square of the distance between them. For particles 1 and 2, with charges q_1 and q_2 at positions $\mathbf{r}_1$ and $\mathbf{r}_2$, the electrostatic force on particle 1 due to particle 2 is given by Coulomb's law

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r_{12}^2}\,\widehat{\mathbf{r}}_{12},$$

where ε_0 is the *permittivity of free space*, r_{12} is the distance between the particles, and $\widehat{\mathbf{r}}_{12}$ is a *unit vector* in the direction of the displacement of particle 1 from particle 2. In order to ensure that this force is not masked by other electric forces caused by *polarization* or *screening* in a medium, it is necessary for the two charged particles to be in a vacuum.

critical angle (B3: 84) When light is incident from one medium, with *refractive index* n_1, onto the boundary of another medium that has a lower refractive index n_2, the maximum angle of incidence for which there is a transmitted beam is known as the critical angle, θ_{crit}, and its value is given by $\sin\theta_{\mathrm{crit}} = n_2/n_1$. For $\theta_{\mathrm{i}} > \theta_{\mathrm{crit}}$, total internal reflection occurs.

critical current (B2: 203) The maximum current that a *superconducting* wire (or other object) can carry with zero *resistance*. When this current is exceeded, the wire reverts to the normal resistive state.

critical magnetic field strength (B2: 201) The field strength above which *superconductivity* is destroyed in a *type-I superconductor*, and the material is in the normal state. *Type-II superconductors* have a lower critical field strength B_{c1} and an upper critical field strength B_{c2}. For $B_{\mathrm{c1}} < B < B_{\mathrm{c2}}$, the material is in the *mixed state*, and the interior of the material becomes normal for $B > B_{\mathrm{c2}}$.

critical temperature (B2: 191) The temperature below which a material becomes a *superconductor*.

cross product An alternative term for a *vector product*.

cross-section (B3: 58) See *scattering cross-section*.

Curie temperature (B2: 48) When the temperature of a *ferromagnetic material* exceeds its Curie temperature, the spontaneous alignment of dipoles within *domains* disappears, and the material becomes *paramagnetic*. Cooling below the Curie temperature restores the ferromagnetic alignment within domains.

curl of a vector field (B1: 110, 233) Given a *vector field* $\mathbf{F}$, the curl of $\mathbf{F}$ is a vector field denoted by $\operatorname{curl}\mathbf{F}$. The *component* of $\operatorname{curl}\mathbf{F}$ in a given direction is the *circulation* per unit area perpendicular to the given direction.

More formally, at any given point, the component of $\operatorname{curl}\mathbf{F}$ in the direction of the unit vector $\widehat{\mathbf{u}}$ is defined

by taking a small plane element centred on the point, with unit normal $\widehat{\mathbf{u}}$. We calculate the circulation of $\mathbf{F}$ round the boundary of this element and divide by the area of the element. The required component of the curl is the limiting value of this ratio as the plane element becomes infinitesimally small.

Curl can also be expressed in terms of *partial derivatives*. In *Cartesian coordinates*,

$$\operatorname{curl}\mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{e}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\mathbf{e}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{e}_z,$$

which may also be written in the form of a *determinant* as

$$\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}.$$

There are also expressions for curl in *cylindrical* and *spherical coordinates*.

curl test (B1: 119, 247) A test used to check whether a given *vector field* is *conservative* or not. The *curl* of the field is calculated. If this is equal to zero throughout a *simply-connected* region, the field is conservative throughout that region. If the curl is non-zero, the field is non-conservative. The curl test can be used to determine whether a given vector field could be an *electrostatic field*.

curl theorem (B1: 110, 234) This theorem states that the *circulation* of a *vector field* $\mathbf{F}$ around a *closed loop* C is equal to the *surface integral* of the *curl* of $\mathbf{F}$, taken over any *open surface* S that is bounded by C. That is,

$$\oint_C \mathbf{F}\cdot \mathrm{d}\mathbf{l} = \int_S (\operatorname{curl}\mathbf{F})\cdot \mathrm{d}\mathbf{S},$$

where C is the perimeter of the open surface S. The curl theorem is also known as *Stokes's theorem*.

curl-free field An alternative term for an *irrotational vector field*.

current Used as a shorthand for *electric current*.

current density (B1: 62) A *vector field* $\mathbf{J}$ which describes the flow of *charge* at each point in space. At a given point, the direction of the current density is the direction in which charge flows, and the *magnitude* of current density is the magnitude of the *current* flowing through a tiny plane element perpendicular to the current flow, divided by the area of the element. The SI unit of current density is $\mathrm{A\,m^{-2}}$.

current dipole (B2: 107) A short, linear current element, the strength of which is given by $I\,\delta l$, where I is the current and δl is the length of the dipole. The SI unit of current dipole strength is A m.

current element (B1: 65) Given a small volume element δV, located at $\mathbf{r}$, the current element associated with this volume element is $\mathbf{J}\,\delta V$, where $\mathbf{J}$ is the *current density*. Given a short directed line segment $\delta\mathbf{l}$ pointing along a wire in the reference direction for current flow, the current element associated with this segment is $I\,\delta\mathbf{l}$, where I is the *current*. The SI unit of current element is A m.

cut-off frequency (B3: 138) The lowest *frequency* at which a particular *mode* will propagate in a *waveguide*.

cut-off wavelength (B3: 138) The longest *wavelength* at which a particular *mode* will propagate in a *waveguide*.

cyclotron (B2: 122) A device for accelerating charged particles to high energies. The particles travel in circular orbits perpendicular to a uniform magnetic field, and their energy is boosted twice in each orbit by an alternating electric field.

cyclotron frequency (B1: 82, B2: 121) When a charged particle moves in a plane perpendicular to a constant uniform *magnetic field*, it performs uniform circular motion. The *angular frequency* of this motion is called the cyclotron frequency and (at non-relativistic speeds) is given by $\omega_\mathrm{c} = |q|B/m$, where q and m are the *charge* and mass of the particle and B is the *magnetic field strength*.

cyclotron motion (B1: 82) The uniform circular motion of a charged particle in a constant uniform *magnetic field*.

cyclotron period (B1: 82, B2: 121) The period of *cyclotron motion*, given by $2\pi/\omega_\mathrm{c}$, where ω_c is the *cyclotron frequency*.

cyclotron radius (B1: 82, B2: 121) The radius of the circular orbit of a charged particle in the plane perpendicular to a magnetic field. The cyclotron radius r_c is related to the particle's charge q, mass m and speed perpendicular to the field $v_\perp$, and the strength B of the magnetic field, by $r_\mathrm{c} = mv_\perp/|q|B$.

cylindrical coordinates (B1: 210) Three coordinates, r, ϕ and z, used to define the position of a point P in three-dimensional space, especially in *axially symmetric* situations.

- The *radial coordinate* r is the perpendicular distance from the z-axis to the point P.
- The *azimuthal coordinate* ϕ is the angle between the x-axis and the line joining the z-axis to the point P. The sense of increasing ϕ is determined by a *right-hand grip rule*: with the thumb of the right hand pointing along the positive z-axis, the curled fingers of the right hand indicate the direction in which ϕ increases.
- The *axial coordinate* z is the usual Cartesian z-coordinate, measured along the z-axis.

The corresponding *Cartesian coordinates* of the point are

$$x = r\cos\phi, \quad y = r\sin\phi \quad \text{and} \quad z = z.$$

Note that, in this course, the angular coordinate ϕ is called the *azimuthal angle*. This ensures that the azimuthal angle has a consistent meaning in cylindrical and *spherical coordinates*. It also means that there is no polar angle in cylindrical coordinates, which is why we avoid the phrase 'cylindrical polar coordinates'.

cylindrical symmetry (B1: 32) An object or a *field* is said to have cylindrical symmetry about a given axis if it is unchanged when rotated through any angle about the axis and is also unchanged when rotated by $180°$ about any axis passing through the mid-point, perpendicular to the axis of symmetry.

cylindrical unit vectors (B1: 211) Three *unit vectors* $\mathbf{e}_r$, $\mathbf{e}_\phi$ and $\mathbf{e}_z$ used to represent a *vector field* in *cylindrical coordinates*. At a given point in space, each unit vector points in a direction where one cylindrical coordinate increases and the other two remain fixed:

- $\mathbf{e}_r$ is in the direction of increasing r and constant ϕ and z. This is the outward radial direction, pointing directly away from the z-axis.
- $\mathbf{e}_\phi$ is in the direction of increasing ϕ and constant r and z.
- $\mathbf{e}_z$ is in the direction of increasing z and constant r and ϕ.

Although $\mathbf{e}_z$ maintains a constant direction, the other two unit vectors vary with position. At a given point, $(\mathbf{e}_r, \mathbf{e}_\phi, \mathbf{e}_z)$ form a right-handed system and the three unit vectors are mutually *orthogonal*.

d'Alembert's solution (B3: 18) The general solution of the *wave equation* in one dimension, which has the form $g_1(z - vt) + g_2(z + vt)$, where g_1 and g_2 are any twice-differentiable functions, and v is the wave speed. This solution corresponds to a wave profile $g_1(u)$ that travels unchanged along the z-axis at speed v in the positive z-direction, and a wave profile $g_2(u)$ that travels unchanged along the z-axis at speed v in the negative z-direction. D'Alembert solutions are only valid if the wave speed is independent of frequency, that is, the material is non-dispersive.

defibrillator (B2: 179) A device used to pass a short pulse of current from a *capacitor* through the human chest to restore normal function of the heart when it is undergoing uncoordinated contractions, known as fibrillations.

del operator (B1: 248) An *operator* defined by

$$\boldsymbol{\nabla} = \frac{\partial}{\partial x}\,\mathbf{e}_x + \frac{\partial}{\partial y}\,\mathbf{e}_y + \frac{\partial}{\partial z}\,\mathbf{e}_z,$$

This operator can act on either a *scalar* or a *vector field*. For example,

$$\operatorname{grad} f = \boldsymbol{\nabla} f, \quad \operatorname{div}\mathbf{F} = \boldsymbol{\nabla}\cdot\mathbf{F}, \quad \operatorname{curl}\mathbf{F} = \boldsymbol{\nabla}\times\mathbf{F}.$$

determinant (B1: 200) A shorthand notation for a particular linear combination of products of quantities. For example, the *vector product* $\mathbf{a} \times \mathbf{b}$ is given by the determinant

$$\begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\,\mathbf{e}_x + (a_z b_x - a_x b_z)\,\mathbf{e}_y + (a_x b_y - a_y b_x)\,\mathbf{e}_z.$$

A determinant may contain a row of operators, e.g. $\partial/\partial x$, etc. and so can also be used to represent the *curl* of a *vector field*.

diamagnetic materials (B2: 46) Such materials exhibit a weak form of magnetism in which *magnetic moments* are induced in atoms in a direction that opposes the applied field. Diamagnetic materials have a small negative *magnetic susceptibility* and a *relative permeability* slightly less than 1. Diamagnetism is temperature-independent. There is a diamagnetic contribution to the *magnetization* in all materials, but in *paramagnetic* and *ferromagnetic* materials, the larger contributions from *permanent magnetic moments* generally dominate the magnetization.

dielectric function (B3: 92) The function that describes the *frequency*-dependence of the *relative permittivity* of a material. The frequency-dependence arises from the dynamics of the interaction of electrons and *permanent electric dipoles* in materials with the electric field of an electromagnetic wave.

dielectric materials (B2: 17) Materials in which the electrons are bound to atoms, and are not free to sustain a steady current. Dielectrics are therefore insulators, in contrast to conducting materials in which charge can flow freely.

differential version of Ampère's law
See *Ampère's law*.

differential version of Faraday's law
See *Faraday's law*.

differential version of Gauss's law
See *Gauss's law*.

differential version of the Ampère–Maxwell law
See *Ampère–Maxwell law*.

differential version of the no-monopole law
See *no-monopole law*.

diffraction grating (B3: 180) An array of features that scatter light coherently as a consequence of their regular spacing. Waves are diffracted at angles θ that satisfy the relationship $n\lambda = d\sin\theta$, where n is an integer, λ is the wavelength and d is the (regular) spacing.

dipolar electric field (B1: 29) The *electric field* produced by an *electric dipole*.

dipolar magnetic field (B1: 75) The *magnetic field* produced by a *magnetic dipole*.

dipole See *electric dipole*, *magnetic dipole*, *current dipole*.

dipole moment See *electric dipole moment* or *magnetic dipole moment*.

dipole potential (B1: 128) An approximation to the *electrostatic potential* of an *electric dipole*, valid in the limit where the distance from the dipole is much greater than the separation of its *charges*.

directed curve (B1: 229) A curve with an associated sense of progression, leading from a start-point to an end-point.

directed line element (B1: 229) A *line element* with an associated sense of progression, leading from a start-point to an end-point. Technically, this differs from a tiny *displacement vector* because it has a definite position in space; a displacement vector does not.

dispersion (B3: 98) The spreading of wave energy in time or space as a consequence of different *frequency* components travelling at different speeds. *Angular dispersion* occurs when light with different frequencies crosses an interface between two materials that have different *refractive indices*. Temporal dispersion occurs when a pulse of radiation travels through a material in which the speed depends on the frequency of the radiation.

dispersion diagram (B3: 98) A graph showing the relationship between *angular frequency* and *wavenumber* for a wave. In the absence of *dispersion*, the graph is linear and passes through the origin, indicating that the wave speed is independent of frequency.

displacement current The displacement current I_d across a given surface S is the *surface integral* of the *displacement current density* $\mathbf{J}_\mathrm{d}$ across the surface:

$$I_\mathrm{d} = \int_S \mathbf{J}_\mathrm{d} \cdot \mathrm{d}\mathbf{S}.$$

displacement current density (B1: 187) A name given to the quantity

$$\mathbf{J}_\mathrm{d} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

that appears in the *Ampère–Maxwell law*. The displacement current density is the *Maxwell term* divided by μ_0, the *permeability of free space*. This terminology is very common, but divides physicists. Many prefer to think of $\mathbf{J}_\mathrm{d}$ as a response of the electromagnetic field, rather than a source, and so regard the term 'displacement current density' as a misnomer.

displacement vector (B1: 193, 197) The displacement vector $\mathbf{r}_\mathrm{AB}$ of point A from point B is the *vector* whose *magnitude* is the distance between A and B and whose direction points towards A from B . So

$$\mathbf{r}_\mathrm{AB} = \mathbf{r}_\mathrm{A} - \mathbf{r}_\mathrm{B},$$

where $\mathbf{r}_\mathrm{A}$ is the *position vector* of the final point and $\mathbf{r}_\mathrm{B}$ is the position vector of the starting point of the displacement.

divergence of a vector field (B1: 56, 224) Given a *vector field* $\mathbf{F}$, the divergence of $\mathbf{F}$ is a *scalar field* denoted by $\operatorname{div} \mathbf{F}$. It represents the *flux* per unit volume of the *vector field*.

More formally, the divergence of $\mathbf{F}$ at a given point P is defined by surrounding P by a small volume element, calculating the flux of $\mathbf{F}$ over the surface of this element and dividing by the volume of the element. The divergence of $\mathbf{F}$ at P is the limiting value of this ratio as the volume element becomes infinitesimally small.

Divergence can also be expressed in terms of *partial derivatives*. In *Cartesian coordinates*

$$\operatorname{div} \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

There are also expressions for divergence in *cylindrical* and *spherical coordinates*.

divergence theorem (B1: 56, 224) This theorem states that the *flux* of a *vector field* $\mathbf{F}$ over a *closed surface* S is equal to the *volume integral* of the *divergence* of $\mathbf{F}$, taken over the volume V inside the closed surface. That is,

$$\int_S \mathbf{F} \cdot \mathrm{d}\mathbf{S} = \int_V \operatorname{div} \mathbf{F} \, \mathrm{d}V,$$

where S is the surface of the volume V. The divergence theorem is also known as Gauss's theorem, which is not to be confused with *Gauss's law*.

divergence-free field (B1: 58) A *vector field* whose *divergence* is equal to zero everywhere.

domain (B2: 47) A small region within a *ferromagnetic material* where the *magnetic dipoles* are all aligned in the same direction. With no external field, the direction of *magnetization* in neighbouring domains will be different, and the net magnetization may be zero. When a magnetic field is applied, domains with a component of magnetization in the field direction grow at the expense of those with a component opposite to the field direction. As the field is increased further, the direction of magnetization within each domain rotates until it is parallel to the applied field; the magnetization is then saturated.

dot product An alternative term for a *scalar product*.

drift speed (B1: 64, B2: 143) The *magnitude* of the *drift velocity* of the charge carriers in an *electric current*.

drift velocity (B1: 64) The average velocity of the charge carriers in an *electric current*.

Earth's electric field (B1: 34) The Earth has an *electric field* at its surface which, on average, has a *magnitude* of $100\,\mathrm{N\,C^{-1}}$ and points vertically downwards. It occurs because the planet's surface carries a *charge* of about -5×10^5 C while the upper atmosphere carries a compensating positive charge.

electric charge (B1: 12) A *scalar* property of certain particles which allows them to exert and respond to *electromagnetic forces*. Electric charge comes in two types — positive and negative. Charges of the same sign repel one another while charges of the opposite sign attract. Charge has some simple properties; see *additivity of charge*, *conservation of charge*, *invariance of charge*, and *quantization of charge*. The SI unit of electric charge is the *coulomb* (C).

electric current (B1: 62) A *scalar quantity* measuring the rate of flow of *charge* through a given surface. The current is positive if the charge flow is in the same general sense as the *unit normals* to the surface; it is negative if the flow is in the opposite sense. The current through a surface is the *surface integral* of the *current density* through the surface. For a current in a wire, the surface is taken to be one that (figuratively) cuts the wire in two. The current through this surface describes the rate of flow of charge in a reference direction along the wire. The SI unit of current is the *ampere* (A).

electric dipole (B1: 29, 127, B2: 18) A stationary pair of *point charges* of equal *magnitudes* and opposite signs separated by a short distance.

electric dipole moment (B1: 128, B2:18) A *vector quantity* expressing the strength and direction of an *electric dipole*. It is defined by

$$\mathbf{p} = q\mathbf{d},$$

where q is the *charge* at the positive end of the dipole and $\mathbf{d}$ is the *displacement vector* from the negative charge to the positive charge. The SI unit of electric dipole moment is C m.

electric displacement (B2: 29) A vector field $\mathbf{D}$ that is related to the *free charge* density ρ_f by Gauss's law: $\operatorname{div}\mathbf{D} = \rho_\mathrm{f}$. For *LIH materials*, $\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}$. More generally, $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$, where $\mathbf{P}$ is the *polarization*. The SI unit of $\mathbf{D}$ is $\mathrm{C\,m^{-2}}$.

electric field (B1: 26) A *vector field* whose value at any given point is the *electric force* per unit charge experienced by a *test charge* placed at the point. That is,

$$\mathbf{E}(\mathbf{r}) = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the electric force experienced by a test charge q at $\mathbf{r}$. The SI unit of electric field is $\mathrm{N\,C^{-1}}$ or $\mathrm{V\,m^{-1}}$. See also *conservative electric field*, *non-conservative electric field*.

electric field line (B1: 28) A continuous directed line drawn in such a way that, at each point along its path, the direction of a field line is the same as the direction of the *electric field*. Electric field lines radiate outwards from positive *charges* and converge inwards towards negative charges. Electric field lines can also extend to infinity. They cannot cross except, possibly, at points where the electric field vanishes. Electric *field line patterns* contain no direct information about the magnitude of the electric field but, as a general rule, the field lines tend to be closer together in regions where the field is stronger.

electric field strength (B1: 26) A positive *scalar quantity* representing the *magnitude* of the *electric field* at a given point.

electric flux (B1: 39) The *surface integral* of the *electric field* over a given surface, S:

$$\text{electric flux} = \int_S \mathbf{E} \cdot \mathrm{d}\mathbf{S}.$$

The SI unit of electric flux is $\mathrm{N\,C^{-1}\,m^2}$.

electric force (B1: 16) The force on a charged particle due to an *electric field*. A particle with charge q at a point where the electric field is $\mathbf{E}$ experiences an electric force $\mathbf{F}$ given by $\mathbf{F} = q\mathbf{E}$. For a stationary *point charge*, the electric force is the total *electromagnetic force* on the charge. If the charge is moving, the electromagnetic force may change, but the electric force at any given point is taken to be independent of the particle's motion.

electric generator (B1: 138) A device which uses *electromagnetic induction* to generate *emfs* and *electric currents*.

electric susceptibility (B2: 23) A measure of the ease with which a material is polarized by an electric field $\mathbf{E}$. It is denoted by χ_E, and defined by the relationship $P = \chi_E\varepsilon_0 E$, where P is the magnitude of the *polarization*. For *LIH materials*, χ_E is independent of the magnitude and direction of $\mathbf{E}$, and independent of position in the material, so $\mathbf{P} = \chi_E\varepsilon_0\mathbf{E}$. The electric susceptibility is a pure number.

electrically isolated A *conductor* surrounded by a vacuum, or by insulating materials, is said to be electrically isolated.

electromagnet (B2: 62) A current-carrying coil designed to produce a magnetic field. Switching

off the current reduces the magnetic field to zero. To generate large fields, the coil is filled with a *ferromagnetic material*.

electromagnetic field *Electric* and/or *magnetic fields*.

electromagnetic force (B1: 12) The force on a charged particle due to other charged particles. In general this includes an *electric force* due to the *electric field* and a *magnetic force* due to the *magnetic field*. The electromagnetic force is one of the four fundamental forces of Nature. It is much stronger than the force of gravity and has a much longer range than nuclear forces. It obeys the *Lorentz force law*.

electromagnetic induction (B1: 138, B2: 154) The creation of an *electric current* or an *emf* by the change in *magnetic flux* through a circuit. The change in flux may be due to a changing *magnetic field* in a stationary circuit, or due to the motion of a circuit in a time-independent non-uniform magnetic field, or due to a combination of both effects.

electromagnetic radiation Another term for *electromagnetic waves*.

electromagnetic wave (B1: 179) A *transverse wave* of *electric* and *magnetic fields*, propagating through space in accordance with *Maxwell's equations*. In empty space, mutually perpendicular electric and magnetic waves remain exactly in phase with one another with $B = E/c$ and travel at the fixed speed

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}},$$

where ε_0 is the *permittivity of free space* and μ_0 is the *permeability of free space*. Light is a type of electromagnetic wave, occupying the small visible part of the electromagnetic spectrum.

electromotance (B1: 143) An alternative term for *emf*.

electromotive force (B1: 142) An alternative term for *emf*. This term is discouraged because electromotive force is not a force in the scientific sense.

electron cyclotron resonance (ECR) (B3: 171) When electrons are exposed to a uniform magnetic field and an electromagnetic wave that propagates parallel to the field, then if the frequency of the wave matches the *cyclotron frequency* of the electrons in that field, resonance occurs and energy is absorbed from the wave. This process is known as electron cyclotron resonance, and it can be used to heat *plasmas*.

electron gun (B2: 119) The combination of a source of electrons and the electrodes that accelerate and focus the electrons into a beam. The source is often a heated electrode — the *cathode*; the accelerating electrode, which is at a positive potential relative to the cathode, is known as the *anode*.

electronvolt (B2: 117) A unit of energy, denoted by eV, equivalent to 1.6×10^{-19} J. It is the change in energy of an electron when displaced through a potential difference of 1 V.

electrostatic field The *electric field* produced by an arrangement of stationary *charges*. An electrostatic field is *conservative*.

electrostatic force (B1: 17) The *electric force* exerted by an arrangement of stationary *charges*.

electrostatic potential (B1: 122) The value of the *electrostatic potential field* at a given point. The SI unit of electrostatic potential is the *volt* (V).

electrostatic potential energy (B2: 169) The potential energy associated with a stationary charged object or system of objects. It is defined as the work done in bringing together the charges, starting with the charges dispersed at infinite separation.

electrostatic potential field (B1: 122) A *scalar field* defined by

$$V(\mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{E} \cdot \mathrm{d}\mathbf{l} + V_0,$$

where $\mathbf{E}$ is an *electrostatic field*, $\mathbf{r}_0$ is an arbitrarily chosen reference point and V_0 is the value of the electrostatic potential at this point. The difference in values of the electrostatic potential between two points is minus the *line integral* of the electrostatic field between the points:

$$V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot \mathrm{d}\mathbf{l}.$$

This definition makes sense for an electrostatic field because, in this case, the line integral depends only on its start-point and end-point. To completely define the electrostatic potential field, its value at a reference point must be fixed. The value at an infinitely distant point is often set equal to zero. Under electrostatic conditions, if we are given an electrostatic potential field, we can immediately deduce the corresponding electrostatic field:

$$\mathbf{E} = -\operatorname{grad} V.$$

electrostatics The study of the *electromagnetic forces* between stationary distributions of *charge*.

emf (B1: 142) If the agencies responsible for maintaining the *current* in a given circuit convert energy δW into electrical form whilst a quantity of *charge* δq is transferred around the circuit, the emf in the circuit is defined to be

$$V_{\text{emf}} = \frac{\delta W}{\delta q}.$$

This is the energy input per unit charge transferred around the circuit. Since *power* is the rate of expenditure of energy, emf is also equal to the power

input per unit current. The SI unit of emf is the *volt*. See also *induced emf*.

energy density (B1: 135, B2: 173) A *scalar field* representing the energy per unit volume associated with a given field. The SI unit of energy density is $\mathrm{J\,m^{-3}}$.

For an *electric field* in empty space, the energy density is

$$u = \frac{1}{2}\varepsilon_0 E^2,$$

where E is the *electric field strength* and ε_0 is the *permittivity of free space*. The total electrostatic energy associated with a particular arrangement of charge is found by integrating the energy density of the electric field over all space.

For a *magnetic field* in empty space, the energy density is

$$u = \frac{1}{2\mu_0}B^2,$$

where B is the *magnetic field strength* and μ_0 is the *permeability of free space*. More generally, in a material where there is an electric field $\mathbf{E}$, an *electric displacement* $\mathbf{D}$, a magnetic field $\mathbf{B}$ and a *magnetic intensity* $\mathbf{H}$, the energy density is

$$u = \tfrac{1}{2}\mathbf{D}\cdot\mathbf{E} + \tfrac{1}{2}\mathbf{B}\cdot\mathbf{H}.$$

energy flux density (B1: 186, B3: 37) The energy passing through unit area normal to the propagation direction in unit time.

equation of continuity (B1: 168) An equation expressing the local *conservation of charge*. It states that

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\mathbf{J} = 0,$$

where ρ is the *charge density* and $\mathbf{J}$ is the *current density*.

equipotential surface (B1: 126) A surface on which the *electrostatic potential* V remains constant. The *electric field line* through any given point is perpendicular to the equipotential surface through that point.

evanescent (B3: 86) A disturbance in space and time that decays with distance but oscillates with time and therefore does not travel like a wave is said to be evanescent. For example, the disturbance $E_0\exp(-z/\delta)\exp(-\mathrm{i}\omega t)$ is evanescent.

event (B2: 220) An occurrence that takes place at a particular instant in time and at a particular point in space. Any event may therefore be uniquely specified by four *coordinates* denoted (ct, x, y, z).

far fields (B3: 51) Those parts of time-dependent magnetic and electric fields that determine the fields far from the source ($r \gg \lambda/2\pi$). These fields are also known as radiation fields; their *amplitude* varies as r^{-1}, so the *power* per unit area they carry varies as r^{-2}.

farad (B1: 133) The SI unit of *capacitance*, equal to one *coulomb* per *volt* ($1\,\mathrm{F} = 1\,\mathrm{C\,V^{-1}}$).

Faraday cage (B1: 121) A conducting cage used to shield the region inside the cage from *electric fields* outside. The cage works well if the mesh size is small enough and if the electric field just outside the cage does not vary too rapidly in time.

Faraday's law (B1: 146, 155) This fundamental law has both integral and differential versions. Either of these is regarded as one of *Maxwell's equations* of electromagnetism.

The integral version of Faraday's law states that

$$\oint_C \mathbf{E}\cdot\mathrm{d}\mathbf{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \mathbf{B}\cdot\mathrm{d}\mathbf{S},$$

where $\mathbf{E}$ is the *electric field*, $\mathbf{B}$ is the *magnetic field*, C is a stationary closed loop, and S is any *open surface* with C as its perimeter. The positive sense of progression around C and the orientation of S are linked by the *right-hand grip rule*. So the *induced emf* around C is equal to the rate of decrease of *magnetic flux* over S.

The differential version of Faraday's law states that

$$\operatorname{curl}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t},$$

where $\mathbf{E}$ is the electric field at a given point and instant, and $\mathbf{B}$ is the magnetic field at the same point and instant.

See also *generalized Faraday law*.

ferromagnetic materials (B2: 47) Such materials exhibit a strong form of magnetism in which the *magnetic dipoles* associated with atoms are aligned within *domains* because of a quantum mechanical coupling between neighbouring dipoles. Ferromagnetic materials have a positive *magnetic susceptibility* χ_B of almost 1 and a *relative permeability* μ much greater than 1. Ferromagnets can be strongly magnetized by weak applied magnetic fields. At the *Curie temperature*, ferromagnetism disappears and the material becomes *paramagnetic*.

fibrils (B3: 175) See *collagen fibrils*.

field (B1: 26, 202) A quantity which, at a given instant, has definite values throughout a region of space. The region may be the whole of space or a continuous set of points within it. It should not be a discrete set of isolated points. If the quantity in question is a *scalar*, the field is a *scalar field*; if the quantity is a *vector*, the field is a *vector field*.

field line of a vector field (B1: 204) A continuous directed line that points in the direction of the *vector field* at each point along its path.

field line pattern (B1: 204) A pattern of *field lines* drawn in a region of space (or a cross-section through the region) showing how the direction of a *vector field* varies with position. A field line pattern contains no direct information about the magnitude of the field.

finite difference form (B2: 87) An approximation to the value of the partial derivative of a scalar function f in terms of the values of the function at neighbouring points on a grid of uniformly-spaced points. For example, the value of $\partial^2 f/\partial x^2$ at the point (x_i, y_j) is approximated by

$$\frac{\partial^2 f}{\partial x^2} \simeq \frac{f(x_{i+1}, y_j) - 2f(x_i, y_j) + f(x_{i-1}, y_j)}{(\Delta x)^2},$$

where $x_{i+1} - x_i = x_i - x_{i-1} = \Delta x$. This approximate form is used in iterative numerical methods for the solution of differential equations.

finite difference method (B2: 87) A method for the numerical solution of a differential equation for a scalar function f in which the region of interest is divided up into a regular grid in two or three dimensions, and simultaneous equations are solved to obtain f at each grid point, given the values of f on the boundary grid points.

flux freezing (B1: 163) The fact that the *magnetic flux* through a closed loop remains practically constant in a very highly conducting fluid, such as a plasma. This is a consequence of *Ohm's law* and the *generalized Faraday law*.

flux of a vector field (B1: 221) Given a plane element that is small enough for the *vector field* $\mathbf{F}$ to be taken as constant all over its surface, the flux of $\mathbf{F}$ over the element is

$$\mathbf{F} \cdot \Delta\mathbf{S} = F_n \, \Delta S,$$

where $\mathbf{F}$ is the field vector on the element, $\Delta\mathbf{S}$ is the *oriented area* of the element, F_n is the normal *component* of the field on the element and ΔS is the area of the element.

The flux of a vector field over an extended surface S is then given by the *surface integral*

$$\text{flux over surface } S = \int_S \mathbf{F} \cdot \mathrm{d}\mathbf{S}.$$

This is interpreted by approximating the surface by many plane elements, adding the fluxes over all these elements and taking the limit as the number of elements increases and their sizes become vanishingly small. See also *additivity of flux*, *electric flux*, *magnetic flux*.

form invariant (B2: 235) An equation is form invariant if it takes the same form in every *inertial frame* of reference, apart from the addition of primes (to indicate transformed quantities) to all non-invariant quantities.

frame of reference (B2: 220) A system of spatial axes and synchronized clocks that will allow an observer to assign a unique set of *coordinates* to each observed *event*.

free charge (B2: 17) The charge in a conducting material associated with the *conduction electrons* that are free to move throughout the material. These electrons carry the electric current. Compare with *bound charge*.

free current (B2: 53) The current in a material that is due to the *conduction electrons*.

free space A term used to describe any region where $\varepsilon = 1$ and $\mu = 1$, so that electromagnetic waves travel at speed $1/\sqrt{\varepsilon\varepsilon_0\mu\mu_0} = 1/\sqrt{\varepsilon_0\mu_0} = c$, the speed of light in a vacuum. Air can be regarded as free space for most purposes.

frequency (B1: 182, B3: 15) The number of cycles per second that a wave undergoes as it passes a given point, or the number of cycles per second of an oscillating system. The frequency f is the reciprocal of the *period* T of the wave:

$$f = \frac{1}{T}.$$

The SI unit of frequency is the hertz (Hz).

Fresnel equations (B3: 81) The equations that define the *amplitude transmission ratios* and *amplitude reflection ratios* for plane electromagnetic waves incident on a *dielectric* boundary.

fundamental frequency (B3: 184) The lowest resonant frequency for an oscillating system.

fundamental mode (B3: 144) The mode in a waveguide with the lowest cut-off frequency.

Gaussian surface (B1: 48) A *closed surface* used to apply the *integral version of Gauss's law*. The Gaussian surface is generally chosen to embody the symmetry of the *charge* distribution, and hence of the *electric field*.

Gauss's law (B1: 41, 56) This fundamental law of electromagnetism has both integral and differential versions. Either of these is regarded as one of *Maxwell's equations* of electromagnetism.

The integral version of Gauss's law states that the *electric flux* over a *closed surface* S is equal to the total *charge* Q enclosed by the surface, divided by ε_0, the *permittivity of free space*. That is,

$$\int_S \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_V \rho(\mathbf{r}) \, \mathrm{d}V,$$

where V is the volume enclosed by S, $\mathbf{E}$ is the *electric field*, and $\rho(\mathbf{r})$ is the *charge density*.

This law applies to all distributions of charge (whether stationary or moving) and to all closed surfaces (no matter what their shape). It is unaffected by the presence of charges outside the closed surface. For the special case of stationary charges, Gauss's law can

be derived from *Coulomb's law*, the *principle of superposition* and the *additivity of charge*.

The differential version of Gauss's law states that the *divergence* of the electric field at a given point and time is equal to the charge density at that point and time, divided by ε_0, the permittivity of free space. That is,

$$\operatorname{div}\mathbf{E} = \frac{\rho}{\varepsilon_0}.$$

It applies to all distributions of charge, whether stationary or moving.

In the presence of dielectric materials, it is often more convenient to express Gauss's law in terms of the *electric displacement* $\mathbf{D}$ and the *free charge* density ρ_f:

$$\int_S \mathbf{D}\cdot\mathrm{d}\mathbf{S} = \int_V \rho_\mathrm{f}\,\mathrm{d}V,$$
$$\operatorname{div}\mathbf{D} = \rho_\mathrm{f}.$$

generalized Faraday law (B1: 162) For a circuit C that moves or distorts in shape, the generalized Faraday law states that

$$\oint_C (\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\mathrm{d}\mathbf{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \mathbf{B}\cdot\mathrm{d}\mathbf{S},$$

where $\mathbf{E}$ is the *electric field*, $\mathbf{B}$ is the *magnetic field*, C is a moving closed loop, and S is any *open surface* with C as its perimeter. The positive sense of progression around C and the orientation of S are linked by the *right-hand grip rule*. So the *induced emf* around C is equal to the rate of decrease of *magnetic flux* over S. Compare with *Faraday's law*.

gradient of a scalar field (B1: 241) Given a *scalar field* f, the gradient of f is a *vector field* denoted by $\operatorname{grad} f$. At any point, its direction is that in which f increases most rapidly and its *magnitude* is the rate of increase of f in this direction. The *component* of $\operatorname{grad} f$ in any given direction is the rate of change of f in that direction.

Gradient can also be expressed in terms of *partial derivatives*. In *Cartesian coordinates*,

$$\operatorname{grad} f = \frac{\partial f}{\partial x}\,\mathbf{e}_x + \frac{\partial f}{\partial y}\,\mathbf{e}_y + \frac{\partial f}{\partial z}\,\mathbf{e}_z.$$

There are also expressions for gradient in *cylindrical* and *spherical coordinates*.

gradient theorem (B1: 244) Given a *scalar field* f, any *line integral* of $\operatorname{grad} f$ can be expressed as the difference between the values of f at the start-point and end-point of the path:

$$\int_{\mathbf{r}_1}^{\mathbf{r}_2} (\operatorname{grad} f)\cdot\mathrm{d}\mathbf{l} = f(\mathbf{r}_2) - f(\mathbf{r}_1).$$

group speed (B3: 106) The speed at which energy and information are transmitted by a wave. It is the speed of motion of the envelope of a packet of waves, and is defined by $v_\mathrm{group} = \mathrm{d}\omega/\mathrm{d}k$. In a non-dispersive medium, $\omega \propto k$, so $v = \omega/k =$ constant, and this constant is just the *phase speed*.

group velocity (B3: 106) The velocity at which energy and information are transmitted by a wave. Often (incorrectly) used when no direction is specified, so that the term *group speed* is more appropriate.

guided wave (B3: 84) An electromagnetic wave that is restricted to a well-defined region and is constrained to follow a required path by conducting or dielectric boundaries. Optical fibre, a pair of parallel conducting planes and a metal tube with rectangular cross-section are examples of *waveguides* that are used for transmitting guided waves.

guided-wave wavelength (B3: 137) The distance along a *waveguide* that spans one complete cycle of the repeating pattern of electric and magnetic fields travelling along the guide in a particular *mode*. It is the shortest distance, along the guide, between two points at which the electric (or magnetic) fields have the same *phase*. The term is also applied to other *guided waves*, such as between two parallel conducting planes.

guided-wave wavenumber (B3: 137) The *wavenumber* for waves in a *waveguide*. If the guided-wave wavenumber is real, the wave propagates along the guide. If it is imaginary, the disturbance is *evanescent*.

Hall effect (B1: 82) When a *magnetic field* is applied perpendicular to a *current*, the *magnetic force* deflects the charge carriers in a direction perpendicular to their motion and to the magnetic field. Surface charges with opposite signs are produced on opposite sides of the conductor, producing an *electric field* (and a *voltage*) across it. This is the Hall effect. In a steady state, the electric and magnetic forces on the charge carriers are equal in magnitude but opposite in direction.

helicon mode (B3: 172) A *right-handed circularly polarized* electromagnetic wave that propagates in a magnetized *plasma* at a frequency below the electron *cyclotron frequency*. Helicon plasma sources are sustained by energy supplied by 13.56 MHz electromagnetic waves that travel as a helicon mode and are absorbed through collisions between electrons accelerated by the wave field and gas molecules.

henry (B2: 159) The SI unit of *self-inductance* L and *mutual inductance* M. It is denoted by the symbol H, and $1\,\mathrm{H} \equiv 1\,\mathrm{T\,m^2\,A^{-1}}$. Since the *induced emf* is given by $V_\mathrm{emf} = -L\,\mathrm{d}I/\mathrm{d}t$, or $V_\mathrm{emf2} = M\,\mathrm{d}I_1/\mathrm{d}t$, a rate of change of current of $1\,\mathrm{A\,s^{-1}}$ will induce an emf of $1\,\mathrm{V}$ in an inductance of $1\,\mathrm{H}$.

Hertzian dipole (B3: 43) A short oscillating *electric dipole* produced by an oscillating current $I_0 \cos \omega t$ flowing through a small length δl. The length δl of a Hertzian dipole is small compared with the *wavelength* of the associated radiation. The *electric dipole moment* varies as $(I_0\, \delta l/\omega) \sin \omega t$.

homogeneous material (B2: 24) The properties of a homogeneous material are the same at all points in the material. In particular, a dielectric or magnetic material is said to be homogeneous if its susceptibility, χ_E or χ_B, is the same at all points in the material.

image charge (B2: 80) Some problems in *electrostatics* involving conducting or dielectric boundaries can be solved by replacing the boundaries with additional charges, such that the boundary conditions for the original problem are satisfied. These additional charges are called image charges and the procedure is known as the *method of images*. The *uniqueness theorem* indicates that the *potential* is the same for the two problems in the region of interest and so also is the *electric field*.

index of refraction (B3: 67) See *refractive index*.

induced electric dipole moment (B2: 19) An *electric dipole moment* in an atom or molecule produced by an external electric field. Induced moments disappear when the field is removed. An applied electric field induces an electric dipole moment in any atom by displacing the centre of the electron cloud by a very small distance from the nucleus.

induced emf (B1: 143) An *emf* produced in a circuit by *electromagnetic induction*. For a stationary circuit C, the induced emf is the *circulation* of the *electric field* $\mathbf{E}$ around C. For a moving circuit, the induced emf is the circulation of $\mathbf{E} + \mathbf{v} \times \mathbf{B}$ around C.

induced magnetic moment (B2: 44) The change in *magnetic dipole moment* of any atom that is the consequence of the orbital motion of the electrons responding to an applied magnetic field. According to *Faraday's law* and *Lenz's law*, the change in current, and therefore the induced moment, is proportional to the applied field and is in the direction that opposes the applied field. The induced moment disappears when the applied field is removed.

inductance See *self-inductance*.

inductor (B2: 163) A coil that is used in an electrical circuit because of its *self-inductance*.

inertial frame of reference (B1: 163, 178, B2: 220) A frame of reference in which all free particles (particles that experience no forces) travel uniformly, with no acceleration. That is, Newton's first law holds true.

instantaneous action at a distance (B1: 25) The idea that the force between two particles is determined by the attributes of the particles and their instantaneous positions (or motions). If instantaneous action at a distance were valid, the force experienced by a particle on Earth due to another particle on the Moon would change instantaneously when the lunar particle is shifted slightly. Information about the shift in position would be communicated instantaneously from the Moon to the Earth. *Coulomb's law* appears to embody instantaneous action at a distance, but action at a distance is never observed in Nature and Coulomb's law is only valid for stationary charges.

insulator A material that does not conduct electric currents.

integral version of Ampère's law
See *Ampère's law*.

integral version of Faraday's law
See *Faraday's law*.

integral version of Gauss's law
See *Gauss's law*.

integral version of the Ampère–Maxwell law
See *Ampère–Maxwell law*.

integral version of the no-monopole law
See *no-monopole law*.

intermediate state (B2: 211) A state in which regions of normal material and *superconductor* coexist for applied field strengths less than the *critical field strength* B_c in a *type-I* superconductor.

invariance of charge (B1: 13) The value of a particle's *charge* is agreed on by all observers. It does not depend on the observer's choice of coordinate system or state of motion. It is also unchanged by *time-reversal*.

invariant (B2: 228) A term used to describe any quantity that takes the same value in all *inertial frames of reference*. The speed of light, c, the *permittivity of free space*, ε_0, the *permeability of free space*, μ_0, and the (rest) mass and charge of a particle are all invariant.

inverse Lorentz transformation (B2: 223) If the *Lorentz transformation* gives the *coordinates* of an *event* in an *inertial frame* $\mathcal{F}'$ in terms of the coordinates in inertial frame $\mathcal{F}$, then the inverse Lorentz transformation gives the event's coordinates in frame $\mathcal{F}$ in terms of the coordinates in frame $\mathcal{F}'$.

inverse problem (B2: 112) A problem in which the cause of an effect must be calculated from measurements of the effect. In magnetism it is straightforward (in principle) to calculate the field produced by a given current distribution. The magnetic inverse problem is to use measurements of magnetic field to determine the current sources that produce the field; this is generally more difficult to do, and unique solutions do not exist.

ionosphere (B3: 156) The upper layers of the Earth's atmosphere, ranging from about $80\,\text{km}$ to about $1500\,\text{km}$, where ultraviolet radiation from the Sun ionizes atoms to create a *plasma*. Long-distance radio transmission is achieved by reflecting radio waves back and forth between the ionosphere and the surface of the Earth.

irrotational vector field (B1: 245) A *vector field* is said to be irrotational in a region R if its *curl* is equal to zero throughout R. Every *conservative* vector field is irrotational. If $\mathbf{F}$ is irrotational in a *simply-connected region*, it is also conservative in that region.

isotropic material (B2: 24) The properties of an isotropic material are the same in all directions. In particular, a *dielectric material* is said to be isotropic if the *electric susceptibility* χ_E is independent of the direction of $\mathbf{E}$, so the *polarization* $\mathbf{P}$ is in the same direction as $\mathbf{E}$. A magnetic material is said to be isotropic if the *magnetic susceptibility* χ_B is independent of the direction of $\mathbf{B}$, so the *magnetization* $\mathbf{M}$ is in the same direction as $\mathbf{B}$.

lamella (B3: 176) A thin sheet. The *stroma* in the *cornea* consists of many lamellae, within each of which *collagen fibrils* are arranged parallel to each other in a matrix of aqueous protein.

Laplace's equation (B2: 72) A second-order partial differential equation for a *scalar field* $f(\mathbf{r})$, given by $\nabla^2 f = 0$, where ∇^2 is the *Laplacian operator*. Laplace's equation is satisfied by the *electrostatic potential* in regions where the *free charge* density is zero.

Laplacian operator (B1: 250, B2: 70) When operating on a *scalar field* f, the Laplacian operator is equivalent to the *gradient* acting on a scalar field f followed by the *divergence*. Also known as the Laplacian, it is denoted by the symbol ∇^2, which is pronounced 'del-squared' or 'nabla-squared'. Thus

$$\nabla^2 f \equiv \operatorname{div}(\operatorname{grad} f).$$

In Cartesian coordinates,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

and there are alternative forms for spherical and cylindrical coordinates given inside the back covers of the course books. The Laplacian operator appears in both *Poisson's equation* and *Laplace's equation*. See also *del operator*.

The Laplacian operator can also operate on a *vector field*. In this case, the same expression is used for ∇^2 in Cartesian coordinates as with scalar fields, but in cylindrical and spherical coordinates the expressions for ∇^2 are much longer and more complicated than those for scalar fields given on the inside back covers of the books.

left-handed (LH) circular polarization (B3: 166) The electric field at a point in the path of an electromagnetic wave that has left-handed circular polarization rotates in the sense given by a left-hand grip rule with respect to the direction of propagation of the wave: with the thumb of the *left* hand in the direction of propagation, the fingers curl in the direction of rotation of the electric field. Note that this is the convention used in plasma physics and electrical engineering; the opposite convention is used in optics.

length contraction (B2: 224) The phenomenon whereby the length assigned to a rod (or any object) depends on the speed of the rod relative to the observer. If l_0 is the length observed in the *inertial frame* in which the rod is at rest, then the length l measured in a frame in which it is moving with constant velocity $\mathbf{v}$ parallel to its length is given by $l = l_0\sqrt{1 - v^2/c^2}$.

Lenz's law (B1: 152) An *induced current* flows in such a way that its *magnetic flux* opposes the change in the magnetic flux that produced it. Lenz's law is consistent with the law of conservation of energy.

LIH material (B2: 24) The acronym LIH refers to materials that are *linear*, *isotropic* and *homogeneous*.

line element A small segment of a straight line.

line integral (B1: 229) The line integral of a *vector field* $\mathbf{F}$ along a given *directed curve* C is written as

$$\int_C \mathbf{F}(\mathbf{r}) \cdot \mathrm{d}\mathbf{l}.$$

To calculate this line integral we approximate C by a set of many *directed line elements* $\Delta\mathbf{l}_1, \Delta\mathbf{l}_2, \ldots, \Delta\mathbf{l}_n$ and form the sum of *scalar products*

$$\sum_i \mathbf{F}_i \cdot \Delta\mathbf{l}_i,$$

where $\mathbf{F}_i$ is the value of the vector field on the ith line element and the sum is taken from $i = 1$ to $i = n$. Taking the limit of an infinite number of infinitesimal line elements gives the required line integral. The units of the line integral are those of the field times length. See also *circulation of a vector field*.

linear material (B2: 24) The response of a linear material to an applied field is proportional to the magnitude of the applied field. In particular, a *dielectric material* is said to be linear if the magnitude P of the *polarization* is directly proportional to the magnitude E of the electric field. A magnetic material is linear if the magnitude M of the *magnetization* is directly proportional to the magnitude B of the magnetic field.

linearity (B3: 18) (of differential equations) A property of many differential equations, including the *wave equation*, whereby a superposition of solutions of the differential equation is also a solution. If $g_1, g_2, g_3, \ldots$ are solutions of a linear differential

equation, then so is $a_1g_1 + a_2g_2 + a_3g_3 + \cdots$, where $a_1, a_2, a_3, \ldots$ are arbitrary constants.

linearly polarized electromagnetic wave (B1: 180) A plane *electromagnetic wave* in which the *electric field* (and hence the *magnetic field*) maintains a fixed alignment. The *polarization direction* is the direction of the electric field and this is always normal to the direction of propagation and normal to the direction of the magnetic field.

London equations (B2: 206) Two equations that were proposed by the London brothers to describe the electrodynamics of *superconductors*, and particularly to be consistent with the *Meissner effect*. One equation relates *current density* $\mathbf{J}_\mathrm{s}$ to electric field in a material with zero *resistance*:

$$\frac{\partial \mathbf{J}_\mathrm{s}}{\partial t} = \frac{n_\mathrm{s}e^2}{m}\mathbf{E},$$

where n_s is the *number density* of *superconducting electrons*, each of which has charge $-e$ and mass m. The other equation relates the current density to the magnetic field, and is more restrictive than *Faraday's law*:

$$\operatorname{curl}\mathbf{J}_\mathrm{s} = -\frac{n_\mathrm{s}e^2}{m}\mathbf{B}.$$

These equations lead to the prediction that both current density and magnetic field fall exponentially with distance inwards from the surface of a superconductor.

Lorentz force law (B1: 80, B2: 117) The *electromagnetic force* acting on a *point charge* q, moving with velocity $\mathbf{v}$, is

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right),$$

where $\mathbf{E}$ is the value of the *electric field* and $\mathbf{B}$ is the value of the *magnetic field* at the position of the charge.

Lorentz transformation (B2: 222) Equations relating the four *coordinates* (ct, x, y, z) of an *event* in one *inertial frame* to the coordinates of the same event in another inertial frame.

lower critical field strength (B2: 213) The magnetic field strength $B_{\mathrm{c}1}$ at which a *type-II superconductor* makes the transition from the completely superconducting state to the *mixed state*.

macroscopic field (B2: 16) The average value of a field over a volume that contains millions of atoms. The averaging process removes information about the atomic scale fluctuations of the field.

magnetar (B1: 78) A type of neutron star that creates huge magnetic fields.

magnetic bottle (B2: 136) A non-uniform magnetic field configuration that can be used to trap charged particles.

magnetic circulation (B1: 92) The magnetic circulation around a given closed loop C is the line integral of the *magnetic field* around C:

$$\text{magnetic circulation} = \oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l}.$$

magnetic dipole (B1: 75, B2: 43) A source of a magnetic field that has a dipolar spatial distribution, similar in form to the field of an *electric dipole* at distances that are not too close to the dipole. A small current loop is a magnetic dipole. In a simple model of an atom, each 'orbiting' electron acts as a magnetic dipole, and electrons have an associated spin that acts as a magnetic dipole. (See *magnetic dipole moment*.)

magnetic dipole moment (B1: 75, B2: 43) A vector quantity that characterizes the strength and direction of a *magnetic dipole*. For a small current loop, the moment $\mathbf{m}$ is given by $\mathbf{m} = |I|\,\Delta\mathbf{S}$, where I is the current and $\Delta\mathbf{S}$ is the oriented area of the loop, with the direction of $\Delta\mathbf{S}$ and $\mathbf{m}$ related to the current circulation by a right-hand grip rule. The magnetic dipole moment of an atom is the vector sum of moments due to the orbits and intrinsic spins of the electrons (plus much smaller contributions from the protons and neutrons). The SI unit of magnetic dipole moment is $\mathrm{A\,m^2}$.

magnetic field (B1: 70, 79) A *vector field* that determines the *magnetic force* on a moving *point charge*, according to the magnetic part of the *Lorentz force law*. The SI unit of magnetic field is the *tesla* (T).

magnetic field line (B1: 73) A continuous directed line drawn in such a way that the direction of the line is the same as the direction of the *magnetic field* at each point along its path. Magnetic field lines do not start or stop at any point in space, but form continuous loops. See the *no-monopole law*.

magnetic field strength (B1: 79) A positive *scalar quantity* representing the *magnitude* of the *magnetic field* at a given point.

magnetic flux (B1: 88) The magnetic flux over a given surface S is the *surface integral* of the *magnetic field* over that surface:

$$\text{magnetic flux} = \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}.$$

The SI unit of magnetic flux is $\mathrm{T\,m^2}$ (also called the weber, Wb). Magnetic flux is often denoted by Φ, the capital Greek letter phi.

magnetic force (B1: 16) The additional *electromagnetic force* that a *point charge* experiences by virtue of being in motion rather than being at rest.

magnetic force law (B1: 71, 79) (1) For a current element: the *magnetic force* on a *current element* $I\,\delta\mathbf{l}$ is

$$\delta\mathbf{F} = I\,\delta\mathbf{l} \times \mathbf{B},$$

where $\mathbf{B}$ is the *magnetic field* at the position of the current element.

(2) For a *point charge*: the magnetic force on a *point charge* q moving with velocity $\mathbf{v}$ is

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B},$$

where $\mathbf{B}$ is the magnetic field at the position of the charge.

magnetic intensity (B2: 54) A vector field $\mathbf{H}$ defined by the relationship $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$, and, for steady fields, related to the *free current* density by $\operatorname{curl} \mathbf{H} = \mathbf{J}_\mathrm{f}$. For *LIH materials*, $\mathbf{H} = \mathbf{B}/\mu\mu_0$, where μ is the *relative permeability* of the material. The SI unit of magnetic intensity is $\mathrm{A\,m^{-1}}$.

magnetic monopole (B1: 88) A hypothetical particle which would act as a source or sink of *magnetic field lines*. In using *Maxwell's equations* we assume that such particles do not exist.

magnetic storm (B1: 153, B2: 136) A small, but sudden and widespread fluctuation in the magnetic field at the Earth's surface caused by an influx to the *Van Allen belts* of charged particles ejected from the Sun. Magnetic storms can induce damaging surges in the grids that distribute electricity across countries and continents.

magnetic susceptibility (B2: 49) A quantity, denoted by χ_B, that relates the *magnetization* $\mathbf{M}$ of a material to the magnetic field $\mathbf{B}$. For *LIH materials*, $\mathbf{M} = \chi_B \mathbf{B}/\mu_0$, and χ_B is independent of the magnitude and direction of $\mathbf{B}$, and also independent of position within the material. *Diamagnetic materials* have a small, negative magnetic susceptibility, *paramagnetic materials* have a small, positive susceptibility, and $\chi_B \simeq 1$ for *ferromagnetic materials*. The magnetic susceptibility is a pure number.

magnetic vector potential (B2: 104) A vector field denoted by $\mathbf{A}$, whose curl gives the magnetic field $\mathbf{B}$, that is $\mathbf{B} = \operatorname{curl} \mathbf{A}$. The SI unit of magnetic vector potential is $\mathrm{T\,m}$.

magnetically-silent source (B1: 106) A current distribution that produces no *magnetic field*.

magnetization (B2: 45) The total *magnetic dipole moment* per unit volume of a material, given by $\mathbf{M} = n\langle \mathbf{m} \rangle$, where n is the *number density* of atoms carrying average magnetic dipole moment $\langle \mathbf{m} \rangle$. Its SI unit is $\mathrm{A\,m^{-1}}$.

magnetization current (B2: 49) The macroscopic current that flows on the surface and within the volume of a magnetized object. Such currents are referred to as *bound currents* — a *bound surface current* and a *bound current density* within the volume of the object — since they are the net effect of the microscopic currents associated with bound electrons in atoms; no electron follows a macroscopic path around the surface or through the volume.

magnetoencephalography (B1: 77, B2: 111) The detection of the tiny *magnetic fields* that are produced by *currents* inside the brain and the attempt to deduce the currents that are responsible from them. This non-invasive technique (often abbreviated to MEG) can track brain activity millisecond by millisecond. It is used for clinical diagnosis and imaging, and for research into brain function.

magnetometer (B2: 111) An instrument for measuring magnetic fields.

magnetosphere (B2: 132, B3: 162) The region in which the Earth's magnetic field has a significant effect on the *solar wind*. It can be regarded as a *plasma*, with a typical number density of charged particles of $10^{11}\,\mathrm{m^{-3}}$.

magnetostatic force (B1: 65) The *magnetic force* between steady, unchanging *currents*. See the *Biot–Savart force law*.

magnitude (B1: 191) The magnitude of a *scalar* Q is a non-negative quantity $|Q|$ describing the size of Q, irrespective of its sign. The magnitude of a *vector* $\mathbf{a}$ is a non-negative quantity $|\mathbf{a}|$, describing the size of $\mathbf{a}$, irrespective of its direction. The magnitude of a vector is usually written as a, omitting both the bold print and the modulus signs. In *Cartesian coordinates* the magnitude of a vector $\mathbf{a}$ is given by

$$a = |\mathbf{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}.$$

Maxwell term (B1: 170) The term $\varepsilon_0\mu_0 \partial\mathbf{E}/\partial t$ that appears in the *Ampère–Maxwell law*.

Maxwell's equations (B1: 177) A set of equations at the heart of the classical theory of electromagnetism. There are four Maxwell equations, each of which can be expressed either in integral or differential versions. These are *Gauss's law*, the *no-monopole law*, *Faraday's law* and the *Ampère–Maxwell law*.

MEG See *magnetoencephalography*.

Meissner effect (B2: 198) One of the defining characteristics of *superconductivity*; the expulsion of magnetic field from a superconductor.

metal detector (B1: 154) A device that uses *electromagnetic induction* to detect metal objects. An oscillating *magnetic field* produced by a transmitting coil induces *currents* in conducting objects. These currents themselves produce oscillating magnetic fields, which are detected by the currents they induce in a separate receiver coil.

method of images (B2: 79) A method for determining the *electrostatic potential* due to a point or line charge in the neighbourhood of a conducting

(or dielectric) boundary. The conductor is replaced by *image charges* which, together with the original charge, produce an electrostatic potential that matches the boundary conditions for the original situation. The *uniqueness theorem* guarantees that this potential will be the complete solution to the original problem.

mixed partial derivative (B1: 215) A second- or higher-order *partial derivative* in which differentiation is taken with respect to two or more different variables. For smoothly-varying functions, the order of partial differentiation does not matter, so you can assume that

$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y}.$$

mixed state (B2: 213) The state of a *type-II superconductor* that exists for field strengths between the *lower* and *upper critical field strengths* $B_{\mathrm{c1}} < B < B_{\mathrm{c2}}$, where the superconducting material is threaded by *normal cores* through which *magnetic flux* passes.

mode (B3: 136) A specific solution to the differential equation describing a system that supports vibrations or waves; different modes are picked out by the geometry of the boundaries.

molecular polarizability (B3: 58) The electron cloud around a molecule is deformed by an external electric field, producing an *electric dipole*. Provided that the field is not too strong, the relationship between *electric dipole moment* $\mathbf{p}$ and electric field $\mathbf{E}$ is linear, with $\mathbf{p} = \varepsilon_0 \alpha \mathbf{E}$, where α is the molecular polarizability.

monochromatic wave (B1: 182, B3: 27) A wave with a single *frequency*. If the wave travels in a vacuum, it also has a single *wavelength*.

mutual inductance (B2: 159) If current I_1 in circuit 1 produces *magnetic flux* Φ_{21} through circuit 2, the mutual inductance M between the circuits is defined as $M = \mathrm{d}\Phi_{21}/\mathrm{d}I_1$. In the absence of ferromagnetic materials, $\Phi_{21} \propto I_1$, so the expression for mutual inductance simplifies to $M = \Phi_{21}/I_1$. Note that $\Phi_{21}/I_1 = \Phi_{12}/I_2$, so the value of the mutual inductance does not depend on which of the two circuits the current flows in. A time-varying current I_1 induces an emf in circuit 2 given by $V_{\mathrm{emf2}} = M\,\mathrm{d}I_1/\mathrm{d}t$. The SI unit of mutual inductance is the *henry* (H).

myelin sheath (B1: 134) A layer of thick insulation around nerve cells. This reduces the *capacitance* of the nerve cell and therefore increases the speed of transmission of nervous impulses along the cell.

natural angular frequency (B2: 187) For an LC circuit, the frequency at which the circuit will oscillate with simple harmonic motion. The natural angular frequency is given by $\omega_{\mathrm{n}} = 1/\sqrt{LC}$. At the natural frequency, the maximum energy stored by the *capacitor* and by the *inductor* are the same, and the energy is transferred back and forth between the two components.

no-monopole law (B1: 89) This fundamental law of electromagnetism has both integral and differential versions. Either of these is regarded as one of *Maxwell's equations* of electromagnetism.

The *integral version* of the no-monopole law states that the *magnetic flux* over any *closed surface* S vanishes:

$$\int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0.$$

The *divergence theorem* then shows that the *divergence* of the *magnetic field* vanishes everywhere:

$$\operatorname{div} \mathbf{B} = 0.$$

This is the *differential version* of the no-monopole law. The no-monopole law would be invalid if *magnetic monopoles* existed.

non-conservative electric field (B1: 115) An *electric field* that has a non-zero *circulation* around a *closed loop*.

normal core (B2: 213) A linear cylindrical region of normal material, parallel to the applied field $\mathbf{B}$, within a *type-II superconductor* in the *mixed state*. The normal cores are arranged in a triangular array, and their number per unit area perpendicular to the field increases as the applied field increases.

normal electrons (B2: 193) In the context of *superconductivity*, normal electrons are the *conduction electrons* that are not condensed into a macroscopic quantum state, and are therefore scattered in the same way as in a normal (i.e. non-superconducting) material.

number density (B1: 64, B2: 23) The number of items per unit volume. Its SI unit is m^{-3}. For example, the number density of atoms or molecules in a gas at $273\,\mathrm{K}$ and 1 atmosphere pressure is about $3 \times 10^{25}\,\mathrm{m}^{-3}$.

observer (B2: 220) In the context of special relativity, an observer can be regarded as an experimenter who records the *coordinates* of *events* using a particular *frame of reference*.

ohm (B1: 145) The SI unit of *resistance* (symbol Ω). A *conductor* has a resistance of 1 ohm if it carries a *current* of 1 *ampere* when a *voltage drop* of 1 *volt* is applied across it.

ohmic conductor A conductor that obeys *Ohm's law*.

Ohm's law (B1: 145, B2: 143) This law states that the *current* I in a *conductor* is proportional to the *voltage drop* V across the conductor:

$$V = IR,$$

where the proportionality constant, R, is the *resistance* of the conductor. Ohm's law is not universally valid, but it is an excellent approximation for many conductors provided that their temperature is held constant. Ohm's law can also be expressed as a local relationship between *current density* $\mathbf{J}$ and *electric field* $\mathbf{E}$ in a material:

$$\mathbf{J} = \sigma\mathbf{E},$$

where σ is the *conductivity* of the material. A more general relationship when the conducting material is moving with velocity $\mathbf{v}$ in magnetic field $\mathbf{B}$ is

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

open surface (B1: 222) A surface that is not a *closed surface*. It is possible to travel from any point in space to any other point without crossing the surface.

operator (B1: 248) Any set of commands that changes a given function into another function. Important operators in electromagnetism include partial differentiation (see *partial derivative*), the *del operator* and the *Laplacian operator*.

oriented area (B1: 39, 220) A plane surface element can be represented by an oriented area. This is a *vector* quantity $\Delta\mathbf{S}$ whose *magnitude* ΔS is the area of the element and whose direction is that of the *unit normal* to the element:

$$\Delta\mathbf{S} = \Delta S\,\widehat{\mathbf{n}}.$$

orthogonal coordinate system (B1: 227) A coordinate system in which the three *unit vectors* are mutually orthogonal.

orthogonal vectors (B1: 198) Two *vectors* are said to be orthogonal if their directions are mutually perpendicular. The *scalar product* of two orthogonal vectors vanishes.

outer core (B1: 78) A spherical shell of swirling liquid iron between 1200 km and 3500 km from the Earth's centre. *Currents* in the outer core are responsible for the Earth's *magnetic field*, which is approximately *dipolar*.

parallel plate capacitor (B1: 54) A *capacitor* consisting of a pair of parallel conducting plates separated by a narrow gap containing a vacuum or an insulating material. The plates normally carry *charges* of equal magnitudes and opposite signs.

paramagnetic materials (B2: 46) Such materials exhibit a weak form of magnetism, due to the partial alignment in an applied magnetic field of permanent *magnetic dipoles* associated with atoms and molecules. In zero applied field, the permanent magnetic dipole moments have random orientations. In an applied field, it is energetically favourable for the dipoles to align themselves with the field direction, but alignment is opposed by random thermal motion. The *magnetization* $\mathbf{M}$ is proportional to the applied field and inversely proportional to the absolute temperature. Paramagnetic materials have a small positive *magnetic susceptibility* and a *relative permeability* that is slightly greater than 1.

partial derivative (B1: 213) Given a function $f(x, y, \ldots)$ of more than one variable, the partial derivative of f with respect to x is the rate of change of f with respect to x when all the other variables are held constant. It is denoted by the symbol

$$\frac{\partial f}{\partial x}.$$

The partial derivative with respect to x is calculated using the standard rules of calculus, remembering that all variables except x must be treated as constants.

Higher-order partial derivatives are formed by a process of successive partial differentiation. For example, a function $f(x, y)$ can be differentiated first with respect to x (holding y constant) and the result can then be differentiated with respect to y (holding x constant). This gives a second-order partial derivative

$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right).$$

See also *mixed partial derivative*.

penetration depth (B2: 208) The characteristic distance that a magnetic field penetrates into a *superconductor*. Denoted by the symbol λ.

Penning trap (B1: 82) A device used to confine charged particles in a vacuum. A strong uniform *magnetic field* pointing in the z-direction is accompanied by a weak *electric field* with a z-component that changes sign at $z = 0$. The *magnetic force* confines particles in the x- and y-directions while the *electric force* confines them in the z-direction.

perfect conductor (B3: 126) A conductor in which there are no collisions between charge carriers and the lattice so that its *resistance* is zero, and its *conductivity* is therefore infinite. A perfect conductor does not exclude magnetic fields. *Superconductors* have zero resistance too, but they have the additional property of perfect *diamagnetism*, and this leads to the exclusion of magnetic fields (the *Meissner effect*).

period (B1: 182, B3: 15) The time interval between successive maxima of a sinusoidal wave at a fixed point in space. The period T is the reciprocal of the *frequency* f, $T = 1/f$.

permanent electric dipole moment (B2: 20) A *dipole moment* in an atom or molecule that has a constant magnitude — it is independent of any applied electric field. An applied field tends to cause a permanent electric dipole moment to align with the field, but this tendency is frustrated by the thermal motion of the atoms.

permanent magnetic moment (B2: 43) The resultant magnetic dipole moment that atoms of many elements possess because the vector sum of the moments due to the orbital motions and spins of all of the electrons is not zero. The magnitude of the moment is independent of applied magnetic field, unlike *induced magnetic moments*. Permanent magnetic moments are responsible for *paramagnetic* and *ferromagnetic* behaviour of materials.

permeability of free space (B1: 67) A fundamental constant $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$ that determines the strength of *magnetic forces*. The proportionality constant in the *Biot–Savart law* is $\mu_0/4\pi$, so the small size of μ_0 reflects the small size of the *magnetostatic force* between two 1 A currents separated by 1 m in vacuum. The permeability of free space also appears in the *Ampère–Maxwell law* so its significance is not limited to magnetostatic situations.

permittivity of free space (B1: 19) A fundamental constant $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$ that determines the strength of *electric forces*. The proportionality constant in *Coulomb's law* is $1/4\pi\varepsilon_0$, so the small size of ε_0 reveals the large *electrostatic force* between two stationary 1 coulomb *charges* separated by 1 metre in a vacuum. The permittivity of free space also appears in two of *Maxwell's equations* (*Gauss's law* and the *Ampère–Maxwell law*) so its significance is not limited to electrostatic situations.

persistent current (B2: 195) A current in a *superconducting* circuit that remains constant as long as the local magnetic field remains constant and the material remains superconducting.

phase (B3: 15) The argument of a periodic function, generally expressed as an angle between zero and 2π (or sometimes $360°$). For the function $\mathbf{E} = \mathbf{E}_0 \sin(kx - \omega t + \phi)$, the phase is $kx - \omega t + \phi$.

phase shift (B3: 15) For a sinusoidal function $A(u) = A_0 \sin(u + \phi)$, the quantity ϕ is called the phase shift. It is equal to the *phase* of the function when $u = 0$.

phase speed (B3: 17) The speed v at which a point of constant *phase* on a sinusoidal wave (such as a wave crest) travels in the direction of propagation. $v = \omega/k = f\lambda$, where ω is the *angular frequency*, k is the *wavenumber*, f is the *frequency*, and λ is the *wavelength*.

pitch angle (B2: 138) The (acute) angle between the direction of motion of a charged particle and the direction of the magnetic field.

plane wave (B1: 180, B3: 22) A *wave* propagating in a fixed direction, with no dependence on the coordinates perpendicular to the direction of propagation. The *phase* has the same value across any infinite plane surface lying perpendicular to the direction of propagation.

plasma (B3: 148) A gaseous condition of matter in which an appreciable fraction of the atoms are ionized. A plasma, which is neutral overall, comprises electrons, ions and neutral atoms.

plasma frequency (B3: 153) The *angular frequency* ω_p below which electromagnetic waves cannot propagate in an *LIH* plasma, owing to the presence of charged particles. Electrons usually dominate owing to their small mass. The plasma frequency is the natural frequency of collective oscillation of free (unbound) electrons about their equilibrium positions in a *plasma*. If the electron number density is n_e, then the plasma frequency is given by $\omega_\mathrm{p} = \left(n_\mathrm{e}e^2/\varepsilon_0 m\right)^{1/2}$.

point charge (B1: 14) A *charged particle* with no internal structure, internal motion or *spin*.

Poisson's equation (B2: 72) A second-order partial differential equation for a *scalar field* $f(\mathbf{r})$, given by $\nabla^2 f = g(\mathbf{r})$, where g is also a scalar field. ∇^2 is the *Laplacian operator*. In a region where the *free charge density* is $\rho_\mathrm{f}(\mathbf{r})$ and the *relative permittivity* is ε, the *electrostatic potential* $V(\mathbf{r})$ satisfies Poisson's equation:

$$\nabla^2 V = -\frac{1}{\varepsilon\varepsilon_0}\rho_\mathrm{f}.$$

When the free charge density is zero, this becomes *Laplace's equation*, $\nabla^2 V = 0$.

polar coordinate (B1: 205) In *spherical coordinates*, the polar coordinate, θ, of a point is the smallest angle measured from the positive z-axis to the line joining the origin to the point.

polarization (B1: 25, B2: 23) (B1) When a foreign *charge* is introduced into a non-conducting medium, small displacements of charge are caused through distortions of electron clouds or re-orientations of molecules. This phenomenon is called polarization. It results in a reduction in the total force between two foreign charges immersed in a non-conducting medium.

(B2) The *electric dipole moment* per unit volume of a *dielectric material*. If the *number density* of dipoles in the material is n, and the average moment of each of the dipoles is $\langle\mathbf{p}\rangle$, then polarization $\mathbf{P} = n\langle\mathbf{p}\rangle$. Its SI unit is $\mathrm{C\,m^{-2}}$.

polarization charge (B2: 25) The macroscopic distribution of charges on the surface and within the volume of a polarized object. These are *bound charges* — a *bound surface charge density* and a *bound volume charge density* within the object — since they are the result of small rearrangements of the charge distribution within atoms and molecules rather than a flow of *free charge* within a conductor.

polarization current density (B2: 40) The current that flows within a *dielectric material* when the

polarization $\mathbf{P}$ changes with time. The polarization current density $\mathbf{J}_\mathrm{p}$ is related to the polarization $\mathbf{P}$ by $\mathbf{J}_\mathrm{p} = \partial\mathbf{P}/\partial t$.

polarization direction (B3: 27) (of an electromagnetic wave) The direction of the electric field vector. In free space, the polarization direction is always transverse to the direction of travel. See also *linearly polarized*, *circularly polarized*.

position vector (B1: 193, 197) A *vector* $\mathbf{r}$ specifying the position of a point P relative to the origin O of coordinates. In *Cartesian coordinates*,

$$\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z,$$

where x, y and z are the Cartesian coordinates of the point. This vector represents the displacement from O to P.

potential (B1: 122) Generally taken as a shorthand for the value of the *electrostatic potential* in situations where the electrostatic context is obvious.

potential difference (B1: 122, 141) The difference in *electrostatic potential* $V(\mathbf{r}_2) - V(\mathbf{r}_1)$ between an end-point $\mathbf{r}_2$ and a start-point $\mathbf{r}_1$. The potential difference is related to a *line integral* of the *electrostatic field* $\mathbf{E}$:

$$V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot\mathrm{d}\mathbf{l}$$

The potential difference is independent of the choice of zero for electrostatic potential. Contrast with *potential* and *potential drop*.

potential drop (B1: 141) The drop in *electrostatic potential* $V(\mathbf{r}_1) - V(\mathbf{r}_2)$ between a start-point $\mathbf{r}_1$ and an end-point $\mathbf{r}_2$. The potential difference is related to a *line integral* of the *electrostatic field* $\mathbf{E}$:

$$V(\mathbf{r}_1) - V(\mathbf{r}_2) = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot\mathrm{d}\mathbf{l}$$

Contrast with *potential difference* and *voltage drop*.

potential field (B1: 122) Generally taken as a shorthand for *electrostatic potential field* in situations where the electrostatic context is obvious.

power (B1: 142) The rate of expenditure of energy. Its SI unit is the watt (W), and $1\,\mathrm{W} \equiv 1\,\mathrm{J\,s^{-1}}$.

Poynting vector (B3: 37) The vector product $\mathbf{N} = \mathbf{E}_\mathrm{phys} \times \mathbf{H}_\mathrm{phys}$ for any electromagnetic wave is called the Poynting vector, where $\mathbf{E}_\mathrm{phys}$ and $\mathbf{H}_\mathrm{phys}$ are the real physical electric and magnetic fields associated with the wave. The Poynting vector represents the *energy flux density* associated with the wave, which is the power per unit area crossing a surface that is perpendicular to the direction of propagation of the wave.

principle of relativity (B2: 222) The principle of relativity asserts that no experiment can distinguish one *inertial frame of reference* from any other identically calibrated inertial frame. An equivalent statement of the principle is that the laws of physics can be written in the same form in all inertial frames.

principle of superposition (B1: 27, 71, 127) For *electric fields*: when there is more than one source of electric field, the total electric field at any point is the *vector sum* of the electric field contributions from all the sources.

For *magnetic fields*: when there is more than one source of magnetic field, the total magnetic field at any point is the vector sum of the magnetic field contributions from all the sources.

For *electrostatic potential*: the electrostatic potential at a given point is equal to the *algebraic sum* of the electrostatic potentials due to all the contributing charges.

propagation vector (B3: 30) A vector $\mathbf{k}$ whose direction is the direction of propagation of a wave and whose magnitude k is the *wavenumber*.

quantization of charge (B1: 14) All isolated particles have *charges* that are integer multiples of e, the charge of a proton. Quarks have charges that are integer multiples of $e/3$, but quarks are never observed as isolated particles.

quantum electrodynamic critical field (B1: 35) According to *quantum electrodynamics*, the insulating nature of the vacuum will break down if a static *electric field* exceeds a critical value throughout a region whose linear dimensions are much larger than 10^{-12} m. The critical value of the field ($1.3 \times 10^{18}\,\mathrm{N\,C^{-1}}$) is called the quantum electrodynamic critical field.

quantum electrodynamics (B1: 35) A modern field theory which combines the theories of electromagnetism and quantum mechanics.

quasistatic (B2: 93) In electromagnetism, the fields, currents and charge densities are said to be quasistatic if they depend on time but change with time slowly enough that the results of *electrostatics* and magnetostatics can still be applied to relate the fields to their sources at each instant of time.

radial coordinate (B1: 205, 210) In *spherical coordinates*, the radial coordinate, r, of a point is the distance of the point from the origin. In *cylindrical coordinates*, the radial coordinate, r, of a point is the distance of the point from the z-axis.

radiation field (B3: 51) At large distances from a small source of electromagnetic waves, the electric and magnetic fields fall off with distance as $1/r$, and in this limit the fields are known as radiation fields. The power crossing unit area is proportional to the square of the fields, so this falls off as $1/r^2$, but since the surface area of a sphere of radius r is proportional

to r^2, the total power crossing a spherical surface is independent of r for radiation fields.

Rayleigh scattering (B3: 57) The scattering of light by molecules or other particles that are small compared with the *wavelength*. The *power* scattered from each molecule is proportional to ω^4, and this accounts for the fact that the sky is blue and sunsets are red.

reflectance (B3: 74) The fraction of the incident *power* of an electromagnetic wave that is reflected from an interface.

reflection rules (B1: 103) (for electromagnetic fields) If an *electric field* is reflected in a plane, the reflected electric field at any point in the plane is obtained by reversing the component of the field perpendicular to the plane, leaving components parallel to the plane unchanged.

If a *magnetic field* is reflected in a plane, the reflected magnetic field at any point in the plane is obtained by reversing components of the field parallel to the plane, leaving the component perpendicular to the plane unchanged.

refractive index (B3: 67) For a *dielectric material* in which the absorption is small, the refractive index is the ratio of the speed of propagation of electromagnetic waves in free space to that in the material. In a dispersive medium, the refractive index depends on *frequency*. When the absorption is significant, the refractive index is a complex quantity and its real part is the ratio of the speed of light in free space to the speed of light in the material. Refractive index n is related to relative permittivity ϵ by $n = \sqrt{\epsilon}$.

relative permeability (B2: 55) For an *isotropic material*, the fields $\mathbf{B}$ and $\mathbf{H}$ are related by $\mathbf{B} = \mu\mu_0\mathbf{H}$, where μ is the relative permeability, which is a pure number. For *diamagnetic materials* μ is slightly less than 1, and for *paramagnetic materials* μ is slightly greater than 1, and in both cases μ is independent of the field. For *ferromagnetic materials*, $\mu \gg 1$, and depends on H.

relative permittivity (B2: 31) For *LIH dielectrics*, the *electric displacement* is related to the electric field by $\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}$, where the quantity ε is known as the relative permittivity of the material. It is related to the *electric susceptibility* χ_E by $\varepsilon = 1 + \chi_E$. Relative permittivity is a pure number. It is also known as the dielectric constant, though it is not a constant since it depends on the material and temperature.

relative permittivity function (B3: 119) A complex function, $\varepsilon_{\text{eff}}(\omega) = \varepsilon - \sigma/\mathrm{i}\omega\varepsilon_0$, that is used in the description of the response of conducting materials to electric fields. It is analogous to the complex *dielectric function* used to describe the response of *dielectric materials*. The Ampère–Maxwell relation can be written in terms of the relative permittivity function as

$$\operatorname{curl}\mathbf{H} = -\mathrm{i}\omega\varepsilon(\omega)\varepsilon_0\mathbf{E}.$$

remanence (B2: 56) The magnitude of the $\mathbf{B}$ field in a *ferromagnetic material* that has been previously magnetized to saturation after the magnetizing field $\mathbf{H}$ has been reduced to zero.

residual current device (B1: 153) A safety device used to cut off *currents* in dangerous situations. In a typical example, the leads bringing current to and from an electrical appliance are near a sensing coil. At a moment of failure, the magnetic flux through the sensing coil changes very rapidly and *electromagnetic induction* creates a current in the coil. This is used to trigger a mechanism that cuts off the current to the appliance.

resistance (B1: 145) A quantity that measures the unwillingness of a conductor to conduct an *electric current*. The resistance R is the proportionality constant in *Ohm's law*, $V = IR$. The SI unit of resistance is the *ohm* (Ω).

resistivity (B2: 142) The resistivity of a material is defined as the reciprocal of the *conductivity* and has the SI unit $\Omega\,\mathrm{m}$.

retarded time (B3: 44) Information takes time r/c to travel from a source to a point a distance r away, so the field at distance r and time t depends on the state of the source at time $t - r/c$, and this is called the retarded time.

return current (B2: 107) The current that flows through the volume of a conducting material between the ends of a current element, such as a *current dipole*. Return currents are needed to complete a circuit and maintain the flow of charge.

right-hand grip rule (B1: 74, 75, 205, 232) A rule used for different purposes:

- To relate the *unit normal* of a surface element to the positive sense of *circulation* around the perimeter of the element: with the thumb of the right hand pointing in the direction of the unit normal to the element, the curled fingers of the right hand indicate the sense of positive circulation around the perimeter of the element.
- To determine the sense of increasing *azimuthal angle*, ϕ, in a *spherical* or *cylindrical coordinate system*: with the thumb of the right hand pointing along the positive z-axis, the curled fingers of the right hand indicate the direction in which ϕ increases.
- For *magnetic field lines* due to a *current element* or a straight current-carrying wire: point your right thumb along the direction of current flow and the curled fingers of your right hand will indicate the sense of circulation of the magnetic field lines.

- For *magnetic dipole moments*: with the fingers of your right hand curled in the direction of current flow, your outstretched right thumb indicates the direction of the magnetic moment.
- For the *magnetic field* at the centre of a current loop: with the fingers of your right hand curled in the direction of current flow, your outstretched right thumb indicates the direction of the magnetic field.

See also *right-hand rule*.

right-hand rule (B1: 194, 199) A rule used for different purposes:

- To determine the direction of a *vector product*: point the fingers of your right hand in the direction of the first *vector*, $\mathbf{a}$, and bend your fingers to point in the direction of the second vector, $\mathbf{b}$. The vector product $\mathbf{a} \times \mathbf{b}$ is then perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, in the direction of your outstretched thumb.
- To determine the handedness of a *Cartesian coordinate system*: point the fingers of your right hand in the direction of the x-axis, and bend them (without dislocation) in the direction of the y-axis. If the outstretched thumb of your right hand points along the z-axis, your coordinate system is *right-handed*; otherwise it is left-handed.

right-handed (RH) circular polarization (B3: 166) The electric field at a point in the path of an electromagnetic wave that has right-handed circular polarization rotates in the sense given by a right-hand grip rule with respect to the direction of propagation of the wave: with the thumb of the *right* hand in the direction of propagation, the fingers curl in the direction of rotation of the electric field. Note that this is the convention used in plasma physics and electrical engineering; the opposite convention is used in optics.

right-handed coordinate system (B1: 194) A *Cartesian coordinate system* chosen in such a way that if you point the fingers of your right-hand in the direction of the x-axis, and bend them (without dislocation) in the direction of the y-axis, the outstretched thumb of your right hand points along the z-axis.

ring current (B2: 135) The current that flows from east to west in the *Van Allen belts* caused by transverse drift of charged particles that are spiralling around the Earth's field lines. Protons drift from east to west, and electrons drift from west to east. The ring current generates a magnetic field at the Earth's surface that opposes the main component of the Earth's field.

scalar (B1: 191) A scalar quantity is one that is fully described by a single number, together with an appropriate unit of measurement.

scalar field (B1: 203) A *field* with *scalar* values.

scalar potential field (B1: 240) If $\mathbf{F}$ is *conservative* in a region R, the scalar potential field is defined by the *line integral*

$$f(\mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{F} \cdot \mathrm{d}\mathbf{l} + f_0,$$

where f_0 is the value of the scalar potential at the reference point $\mathbf{r}_0$, and the path of the line integral is entirely within R.

The *conservative vector field* $\mathbf{F}$ is related to the *gradient* of the scalar potential field f:

$$\mathbf{F} = -\operatorname{grad} f,$$

and any line integral of $\mathbf{F}$ is related to a difference in function values of f:

$$\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot \mathrm{d}\mathbf{l} = -\Big(f(\mathbf{r}_2) - f(\mathbf{r}_1)\Big).$$

See also *electrostatic potential field*.

scalar product (B1: 197) The scalar product of any two *vectors* $\mathbf{a}$ and $\mathbf{b}$ is a *scalar quantity* defined by

$$\mathbf{a} \cdot \mathbf{b} = ab\cos\theta,$$

where a and b are the *magnitudes* of the vectors and θ is the smaller of the angles between their directions. An equivalent definition can be given in terms of the *Cartesian components* of the vectors:

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z.$$

scale factor (B1: 209) In an *orthogonal coordinate system*, suppose a coordinate q (which could be an angle) changes by a small amount δq. Then the corresponding *displacement vector* is $h_q\, \delta q\, \mathbf{e}_q$, where h_q is the scale factor of the coordinate and $\mathbf{e}_q$ is the *unit vector* corresponding to increasing q.

scaling a vector (B1: 192) The process of multiplying a *vector* $\mathbf{a}$ by a *scalar* λ to create a new vector $\lambda\mathbf{a}$. The new vector has *magnitude* $|\lambda|a$ and points in the direction of $\mathbf{a}$ if λ is positive and in the opposite direction if λ is negative. In terms of components,

$$\lambda\mathbf{a} = \lambda a_x \mathbf{e}_x + \lambda a_y \mathbf{e}_y + \lambda a_z \mathbf{e}_z,$$

so each *Cartesian component* of the vector is multiplied by λ.

scattering cross-section (B3: 58) A measure of the *power* scattered from a beam of incident radiation by a scattering object relative to the incident power per unit area. It is represented by the symbol σ, and its SI unit is m^2. In the case of scattering of light by a molecule, if the incident power per unit area is $\overline{N}$ and the scattered power from a molecule is $\overline{W}$, then $\sigma = \overline{W}/\overline{N}$.

scattering length (B3: 60) When a beam of radiation is scattered by molecules or other particles, the *power* per unit area, N, in the beam falls exponentially with distance z travelled according to the relationship $N = N_0 \exp(-z/L_{\mathrm{scat}})$. The quantity L_{scat} is the scattering length, and is the

distance over which the power decreases by a factor of $\mathrm{e}^{-1} = 0.37$. It is related to the *number density* n of scatterers and their *scattering cross-section* σ by $L_{\mathrm{scat}} = 1/n\sigma$.

scattering plane (B3: 76) When an electromagnetic wave is incident on a boundary between two materials, the wave vectors for the incident, reflected and transmitted waves all lie in a plane that is normal to the boundary, and this plane is known as the scattering plane.

screening (B1: 24) When a foreign *charge* is introduced into a *conductor* containing mobile charges, it becomes surrounded by charges of the opposite sign to its own. This phenomenon is known as screening. It causes the effective force between the two foreign charges to fall off very rapidly with separation.

self-inductance (B2: 162) If current I in a circuit produces *magnetic flux* Φ through the circuit, the self-inductance L of the circuit is defined as $L = \mathrm{d}\Phi/\mathrm{d}I$. In the absence of *ferromagnetic materials*, $\Phi \propto I$, so the expression for self-inductance simplifies to $L = \Phi/I$. The term self-inductance is often abbreviated to inductance. A time-varying current I in a circuit induces an *emf* $V_{\mathrm{emf}} = -L\,\mathrm{d}I/\mathrm{d}t$. The SI unit of inductance is the *henry* (H).

simply-connected region (B1: 246) A region in which any *closed loop* can be continuously distorted and shrunk to a point without leaving the region.

skin depth (B3: 125) The distance that characterizes the exponential attenuation of the *amplitude* of an electromagnetic wave as it penetrates a conductor. The skin depth δ is given by

$$\delta = \sqrt{\frac{2}{\mu_0\sigma\omega}},$$

where σ is the *conductivity*, ω is the *angular frequency* and μ_0 is the *permeability of free space*. The significance of the skin depth is that the amplitudes of the fields at distance δ below the surface are smaller by a factor of $1/\mathrm{e} \simeq 0.37$ than the amplitudes at the surface.

skin effect (B3: 126) At high frequencies, the current in a conductor is confined to a thin outer layer of the material that has thickness of the order of the *skin depth*. This is known as the skin effect.

Snell's law (B3: 75) The relationship between the angle of incidence, θ_{i}, for electromagnetic radiation striking a boundary between two dielectric media and the angle of transmission (or refraction), θ_{t}, is known as Snell's law: $\sin\theta_{\mathrm{t}}/\sin\theta_{\mathrm{i}} = n_1/n_2$, where n_1 and n_2 are the *refractive indices* of the first (incident) and second (transmitted) media.

solar wind (B2: 132) A continuous (though not constant) stream of charged particles, mostly protons and electrons, emanating from the Sun. The solar wind has a major influence on the Earth's magnetic field at distances more than a few Earth radii from the Earth.

solenoid (B1: 106) A conducting helical coil whose densely-packed turns are wrapped uniformly around the surface of a cylinder. The *magnetic field* produced by a long solenoid is almost uniform inside the solenoid, except at points that are close to its ends.

solenoidal field An alternative term for a *divergence-free* field.

special relativity (B2: 221) A theory of relativity based on the *principle of relativity* and the constancy of the speed of light in a vacuum. The theory is essentially restricted to the special case of observers in *inertial frames of reference* that are moving with constant relative velocity; hence the use of the term 'special'.

spherical coordinates (B1: 205) Three coordinates, r, θ and ϕ, used to define the position of a point P in three-dimensional space, especially in spherically symmetric situations.

- The *radial coordinate* r is the distance from the origin O to the point P.
- The *polar coordinate* θ is the angle measured from the positive z-axis to the line OP.
- The *azimuthal coordinate* ϕ is the angle between the x-axis and the projection of OP in the xy-plane. The sense of increasing ϕ is determined by a *right-hand grip rule*: with the thumb of the right hand pointing along the positive z-axis, the curled fingers of the right hand indicate the direction in which ϕ increases.

The corresponding *Cartesian coordinates* of the point are:

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi \quad \text{and} \quad z = r\cos\theta.$$

spherical symmetry (B1: 30) An object or a *field* is said to be spherically symmetric about a given point if it is unchanged when rotated through any angle about any axis through the point.

spherical unit vectors (B1: 206) Three *unit vectors* $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ used to represent a *vector field* in *spherical coordinates*. At a given point in space, each unit vector points in a direction where one spherical coordinate increases and the other two remain fixed:

- $\mathbf{e}_r$ is in the direction of increasing r and constant θ and ϕ. This is the outward radial direction, pointing directly away from the origin.
- $\mathbf{e}_\theta$ is in the direction of increasing θ and constant r and ϕ.
- $\mathbf{e}_\phi$ is in the direction of increasing ϕ and constant r and θ.

All three unit vectors vary with position. At a given point, $(\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi)$ form a right-handed system and the three unit vectors are mutually *orthogonal*.

spherical wave (B3: 51) A wave for which the *phase* of the quantity that varies ($\mathbf{E}$ and $\mathbf{B}$ for an electromagnetic wave) at an instant of time has the same value over the surface of a sphere centred on the source of the waves. In the absence of *absorption*, the *amplitude* of the wave decreases as r^{-1}, and the *power* per unit area decreases as r^{-2}.

spin (B1: 14) An intrinsic property of electrons and other elementary particles that produces magnetic effects.

SQUID magnetometer (B2: 111) The most sensitive instrument for measuring small magnetic fields. It incorporates a Superconducting Quantum Interference Device, or SQUID, the operation of which depends on quantum properties of superconducting circuits. The maximum achievable sensitivity is about 10^{-15} T.

standard configuration (B2: 221) Two *inertial frames of reference*, $\mathcal{F}$ and $\mathcal{F}'$, are in standard configuration if their corresponding axes are parallel, the origin of $\mathcal{F}'$ moves at constant velocity along the x-axis of $\mathcal{F}$, and, in addition, the *event* at which the origins of the frames coincide occurs at time $t = 0$ in $\mathcal{F}$ and at time $t' = 0$ in $\mathcal{F}'$.

standing wave (B3: 134) The superposition of a wave incident on a boundary and the reflected wave creates a standing wave. Standing waves have nodes — positions of zero displacement — at fixed points separated by half of the wavelength, and midway between the nodes are antinodes, where the displacement oscillates with maximum amplitude. Alternate antinodes have a phase difference of π. For an electromagnetic standing wave formed when a wave is reflected at normal incidence from a conducting boundary, the electric field standing wave is $\pi/2$ out of phase with the magnetic field standing wave, and also shifted in space by $\lambda/4$ relative to the magnetic field standing wave.

steady field A field that does not vary with time.

Stokes's theorem (B1: 234) Another term for the *curl theorem*.

stroma (B3: 176) Generally, the tissue that forms the framework of an organ. In the case of the *cornea*, the stroma is the middle layer, which makes up 90% of its thickness and is sandwiched between special surface layers. The stroma consists of about 250 *lamellae*, within each of which there are *collagen fibrils* arranged approximately parallel to each other and to the surface of the lamella.

sunspots (B1: 78) Regions of particularly high *magnetic field* on the Sun's surface. Sunspots are cooler and darker than the surrounding solar surface because their high magnetic fields inhibit hot gases from rising to the surface.

superconducting electrons (B2: 193) In the BCS theory, those electrons that are in a macroscopic quantum state that extends throughout the *superconductor*. At absolute zero, all of the *conduction electrons* are superconducting, but the proportion that are superconducting decreases with increasing temperature (and increasing magnetic field) to zero at the *critical temperature*.

superconductor (B2: 191) A material in which there is zero *resistance* to the flow of a steady electric current. An important property of superconductors is the *Meissner effect* — the exclusion of magnetic field $\mathbf{B}$ from their interior; superconductors are said to be perfectly *diamagnetic*.

surface charge density (B1: 54) The *charge* per unit area around a given point on a surface. The SI unit of surface charge density is $\mathrm{C\,m^{-2}}$.

surface integral of a vector field (B1: 222) The surface integral or *flux* of a *vector field* $\mathbf{F}$ over a surface S is written as

$$\int_S \mathbf{F}(\mathbf{r}) \cdot \mathrm{d}\mathbf{S}.$$

To calculate this surface integral we approximate S by many small plane elements, with the *unit normals* of neighbouring elements taken to be almost parallel (rather than almost antiparallel). For any *closed surface*, it is conventional to choose the unit normals to point outwards into the exterior space. We then form the sum

$$\sum_i \mathbf{F}_i \cdot \Delta\mathbf{S}_i,$$

where $\mathbf{F}_i$ is the value of the vector field on the ith surface element and the sum extends over all elements on the surface S. Taking the limit of an infinite number of infinitesimal surface elements gives the required surface integral (or flux). For a closed surface, this is the flux across the surface into the exterior space.

surface magnetization current See *bound surface current*.

susceptibility See *electric susceptibility*, *magnetic susceptibility*.

symmetry principle (B1: 30, 101) for *electromagnetic fields*. This principle states that any operation that leaves the source of an electromagnetic field unchanged also leaves the field unchanged.

TE mode (B3: 138) An abbreviation for transverse electric mode. A *mode* for a *guided wave* in which the electric field does not have a component in the direction of propagation of the wave, but the magnetic field does.

TE wave (B3: 138) An abbreviation for transverse electric wave. A guided electromagnetic wave in which the electric field does not have a component in the direction of propagation of the wave, but the magnetic field does.

TEM wave (B3: 182) An abbreviation for transverse electric and magnetic wave. A term used for guided electromagnetic waves that have both electric and magnetic fields transverse to the direction of propagation. 'Unguided' waves in free space or in media are TEM waves, but the term is generally used only in the context of *guided waves*.

tesla (B1: 71) The SI unit of *magnetic field* (symbol T). If a *point charge* of 1 *coulomb* moves at a speed of 1 metre per second perpendicular to a *magnetic field* of magnitude 1 *tesla*, it experiences a *magnetic force* of 1 newton.

test charge (B1: 26) A *charge* used to test the value of an *electric field*. The test charge is generally taken to be small enough to cause a negligible disturbance of the sources of the field.

time constant (B2: 176) The time taken for a variable to decay to $1/\mathrm{e}$ of its initial value. For a circuit containing resistance R and capacitance C, the time constant is RC. For a circuit containing resistance R and inductance L, the time constant is L/R.

time dilation (B2: 225) The phenomenon whereby the time interval between two ticks of a clock depends on the *inertial frame* in which the clock is observed. If τ_0 is the time interval observed in an inertial frame in which the clock is at rest, then the time interval, τ, observed in an inertial frame in which the clock is moving with speed v is given by $\tau = \tau_0/\sqrt{1 - v^2/c^2}$.

time-reversal symmetry (B1: 32) In electromagnetism, time-reversal symmetry refers to the fact that *electromagnetic forces* are unchanged by a reversal of the flow of time. Taking *electric charge* to be unchanged by time-reversal, it follows that *electric fields* are unchanged by time-reversal but magnetic fields are reversed by time-reversal.

TM mode (B3: 139) An abbreviation for transverse magnetic mode. A *mode* for a *guided wave* in which the magnetic field does not have a component in the direction of propagation of the wave, but the electric field does.

TM wave (B3: 139) An abbreviation for transverse magnetic wave. A guided electromagnetic wave in which the magnetic field does not have a component in the direction of propagation of the wave, but the electric field does.

toroidal solenoid (B1: 108) A conducting coil whose densely-packed turns are wrapped uniformly around the surface of a torus.

transcranial magnetic stimulation (B1: 154) A medical technique which uses a rapidly alternating *magnetic field* to induce currents in the brain.

transformer (B1: 153) A device that uses *electromagnetic induction* to increase or decrease alternating voltages. Two coils containing different numbers of turns are magnetically coupled, so that changes in the *magnetic flux* through a primary coil cause similar (but enhanced or reduced) changes in the magnetic flux through a secondary coil.

translational symmetry (B1: 32) An object has translational symmetry along a fixed axis if it is unchanged by any displacement along the axis.

translucence (B3: 176) The incoherent transmission of light. Wavefronts are distorted by transmission through a translucent material, like a pane of frosted glass, so overall patterns of light and dark can be seen, but not a detailed image.

transmittance (B3: 74) The fraction of the incident power of an electromagnetic wave that is transmitted across an interface.

transparency (B3: 176) The property of coherent transmission of light. You can see clearly the details of an image through a transparent material, like a normal pane of glass.

transverse electric wave (B3: 138) See *TE wave*.

transverse magnetic wave (B3: 139) See *TM wave*.

transverse wave (B1: 180, B3: 10) A *wave* in which the physical property of interest oscillates perpendicular to the direction of propagation of the wave. The electric wave and magnetic wave are both transverse in an *electromagnetic wave* in *free space* or in bulk dielectric media. In contrast, for longitudinal waves, such as sound waves, the physical property of interest oscillates in the direction of propagation of the wave. Some *guided waves* are not transverse, since they have electric or magnetic fields with longitudinal components, in the direction of propagation, as well as transverse components.

triangle rule (B1: 193) A geometrical rule for adding two *vectors*. Arrows representing the two vectors are drawn with the head of the first arrow, $\mathbf{a}$, coincident with the tail of the second arrow, $\mathbf{b}$. The arrow joining the tail of $\mathbf{a}$ to the head of $\mathbf{b}$ then represents the vector sum $\mathbf{a} + \mathbf{b}$.

two-fluid model (B2: 204) A model for a *superconductor* that treats the *free electrons* as two fluids. One fluid consists of *normal electrons*, and these behave in the same way as the free electrons in a normal metal. The other consists of *superconducting electrons*, which are not scattered and can flow without resistance.

type-I superconductor (B2: 210) All of the pure

elemental *superconductors* are type-I superconductors, with the exception of niobium, vanadium and technetium. Type-I superconductors have a low *critical temperature* T_c, and when a *magnetic field* is applied parallel to a thin specimen, the superconductivity is destroyed above a well-defined *critical magnetic field strength* B_c. For thick specimens, an *intermediate state* comprising regions of normal and superconducting material exists for a range of field strengths below the critical field strength. In type-I materials, the *penetration depth* is smaller than the *coherence length*, and the surface energy, associated with the boundary between superconducting and normal regions, is positive.

type-II superconductor (B2: 210) All *superconductors* are type-II, except for the elements, but including niobium, vanadium and technetium. Type-II superconductors expel and exclude magnetic fields below their *lower critical field strength* B_{c1}, but in the range from B_{c1} to the *upper critical field strength* B_{c2}, the material is in the *mixed state*, in which it is threaded by thin regions of normal material. In type-II materials, the *penetration depth* is greater than the *coherence length*, and the surface energy, associated with the boundary between superconducting and normal regions, is negative.

uniform field A *field* that does not vary in space.

uniqueness theorem (B2: 79) In a region where *Poisson's* or *Laplace's equation* applies and the value of the *potential* V is known at all points on the boundary, then there is only one possible solution of the equation for V in the region.

unit normal (B1: 220) A *unit vector* that is normal to a given surface at a given point. On a *closed surface*, the unit normals are taken to point outwards into the exterior space. On an *open surface*, selection of a unit normal involves an arbitrary choice of one of the two possible directions normal to the surface.

unit vector (B1: 193) A *vector* of *magnitude* 1 (with no units). Every non-zero vector $\mathbf{a}$ has a corresponding unit vector $\widehat{\mathbf{a}}$ which points in the same direction as $\mathbf{a}$, but has unit magnitude. Unit vectors are used to define the directions of Cartesian coordinate axes (see *Cartesian unit vectors*), to define directions in which only one coordinate varies in more general coordinate systems (see *spherical unit vectors* and *cylindrical unit vectors*) and to define a direction normal to a surface (see *unit normal*).

upper critical field strength (B2: 213) The *magnetic field strength* B_{c2} above which the *mixed state* in a *type-II superconductor* ceases to exist and the bulk of the material becomes normal.

Van Allen belts (B2: 130) Regions surrounding the Earth where charged particles (mainly electrons and protons) are trapped by the Earth's magnetic field.

vector (B1: 191) A vector quantity is one that is fully characterized by its magnitude and direction. All vectors (except the *zero vector*) point in a definite direction in space.

vector calculus identity (B1: 249) A relationship involving *gradients*, *divergences* or *curls* that is valid for all appropriate fields in all coordinate systems.

vector field (B1: 203) A *field* with *vector* values.

vector identity (B1: 202) A relationship between *vectors* that is valid no matter which vectors are chosen.

vector potential (B2: 104) An abbreviation for *magnetic vector potential*.

vector product (B1: 199) The vector product of any two *vectors* $\mathbf{a}$ and $\mathbf{b}$ is a vector quantity defined by

$$\mathbf{a} \times \mathbf{b} = ab \sin\theta\, \widehat{\mathbf{n}},$$

where a and b are the *magnitudes* of the vectors and θ is the smaller of the angles between their directions. The *unit vector* $\widehat{\mathbf{n}}$ is normal the plane of $\mathbf{a}$ and $\mathbf{b}$ pointing in a direction determined by the *right-hand rule*. An equivalent definition can be given in terms of the *components* of the vectors in a *right-handed Cartesian coordinate system*:

$$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y)\mathbf{e}_x + (a_z b_x - a_x b_z)\mathbf{e}_y + (a_x b_y - a_y b_x)\mathbf{e}_z.$$

This can also be represented by a *determinant*:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}.$$

vector sum (B1: 193) A sum of *vectors*, with due account taken of their directions.

volt (B1: 123) The SI unit of *potential* and *potential difference* (symbol V). One volt is equal to one joule per *coulomb* ($1\,\mathrm{V} = 1\,\mathrm{J\,C^{-1}}$).

voltage drop (B1: 142, 159) The voltage drop along a path C that is fixed in space is

$$V_{\text{drop}} = \int_C \mathbf{E} \cdot \mathrm{d}\mathbf{l},$$

where $\mathbf{E}$ is the *electric field* at the position of an element $d\mathbf{l}$.

The voltage drop along a path C that is moving through space is

$$V_{\text{drop}} = \int_C (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l},$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields at the position of an element $d\mathbf{l}$ that has velocity $\mathbf{v}$. The SI unit of voltage drop is the *volt*.

volume integral (B1: 217) The *volume integral* of a scalar field f over a region R is written as

$$\int_R f(\mathbf{r})\,\mathrm{d}V.$$

The volume integral is calculated by dividing R into many small volume elements and forming the sum

$$\sum_i f_i \, \Delta V_i,$$

where f_i is the value of the field inside the ith volume element and the sum extends over all volume elements in the region R. Taking the limit of an infinite number of infinitesimal volume elements gives the required volume integral.

To evaluate a volume integral, it is wise to take account of the symmetry of the region of integration of the function that is being integrated. For example, *spherical symmetry* prompts the use of *spherical coordinates* and *axial symmetry* prompts the use of *cylindrical coordinates*.

volume magnetization current See *bound current density*.

wave A disturbance that oscillates in time and propagates through space.

wave equation (B3: 14) A second-order partial differential equation linking derivatives in time and space. In one dimension, the wave equation for a quantity A is

$$\frac{\partial^2}{\partial z^2} A(z,t) = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} A(z,t),$$

and its solutions correspond to propagation of the quantity A through space as waves with speed v.

wavefront (B3: 22) A surface on which the *phase* of a wave is constant at a particular instant of time.

waveguide (B3: 140) A structure used to confine electromagnetic waves and guide them between two points. Optical fibre and rectangular metal tubes are examples of waveguides.

waveguide mode (B3: 141) A particular pattern of electric and magnetic fields travelling in a waveguide. Each mode represents a solution of the wave equations that satisfies the boundary conditions on the walls of the guide.

wavelength (B1: 182, B3: 15) The distance λ between successive maxima of a sinusoidal wave, at a fixed instant of time. More generally, the distance between neighbouring points that have the same *phase*.

wavenumber (B1: 182, B3: 15) The wavenumber k of a *monochromatic wave* is related to its *wavelength* λ by

$$k = \frac{2\pi}{\lambda}.$$

The SI unit of wavenumber is m^{-1}.

whistler waves (B3: 164) Audio frequency electromagnetic radiation in the magnetosphere, originating from the electrical noise of lightning strokes. The disturbances travel along the Earth's magnetic field direction, and lower-frequency components travel more slowly than higher-frequency components, distorting the original crack of a lightning stroke into a sound that descends through the audio range over a few seconds.

work–energy theorem (B1: 125) This states that, when a particle is acted on by a *conservative* force, the change in its kinetic energy is the total work done by the force, obtained by evaluating its *line integral* along the path of the particle:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot \mathrm{d}\mathbf{l}.$$

zero vector (B1: 192, 196) A *vector* with zero *magnitude* denoted by the bold symbol $\mathbf{0}$. In terms of *Cartesian components*,

$$\mathbf{0} = 0\mathbf{e}_x + 0\mathbf{e}_y + 0\mathbf{e}_z.$$

Uniquely among vectors, the zero vector does not point in any definite direction.

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